

Refer to the notes at
the end of the Table

FUNCTION-TRANSFORM PAIRS

No.	$F(s)$	$f(t), t \geq 0$
1	1	$\delta(t)$
2	$\frac{1}{s}$	$U(t)$
3	$\frac{1}{s^n} \ (n = 1, 2, 3, \dots)$	$\frac{t^{n-1}}{(n-1)!}$
4	$\frac{1}{s+a}$	ϵ^{-at}
5	$\frac{1}{(s+a)(s+b)}$	$\frac{1}{b-a} (\epsilon^{-at} - \epsilon^{-bt})$
6	$\frac{s}{(s+a)(s+b)}$	$\frac{1}{a-b} (a\epsilon^{-at} - b\epsilon^{-bt})$
7	$\frac{1}{(s+a)(s+b)(s+c)}$	$\frac{\epsilon^{-at}}{(b-a)(c-a)} + \frac{\epsilon^{-bt}}{(a-b)(c-b)}$ $+ \frac{\epsilon^{-ct}}{(a-c)(b-c)}$
8	$\frac{s}{(s+a)(s+b)(s+c)}$	$-\frac{a\epsilon^{-at}}{(b-a)(c-a)} - \frac{b\epsilon^{-bt}}{(a-b)(c-b)}$ $- \frac{c\epsilon^{-ct}}{(a-c)(b-c)}$
9	$\frac{s^2}{(s+a)(s+b)(s+c)}$	$\frac{a^2\epsilon^{-at}}{(b-a)(c-a)} + \frac{b^2\epsilon^{-bt}}{(a-b)(c-b)}$ $+ \frac{c^2\epsilon^{-ct}}{(a-c)(b-c)}$
10	$\frac{1}{(s+a)^2}$	$t\epsilon^{-at}$
11	$\frac{s}{(s+a)^2}$	$(1-at)\epsilon^{-at}$
12	$\frac{1}{(s+a)(s+b)^2}$	$\frac{1}{(a-b)^2} \epsilon^{-at} + \frac{(a-b)t-1}{(a-b)^2} \epsilon^{-bt}$
13	$\frac{s}{(s+a)(s+b)^2}$	$-\frac{a}{(a-b)^2} \epsilon^{-at} - \frac{b(a-b)t-a}{(a-b)^2} \epsilon^{-bt}$
14	$\frac{s^2}{(s+a)(s+b)^2}$	$\frac{a^2}{(a-b)^2} \epsilon^{-at}$ $+ \frac{b^2(a-b)t+b^2-2ab}{(a-b)^2} \epsilon^{-bt}$

15	$\frac{1}{(s+a)^n}$	$\frac{1}{(n-1)!} t^{n-1} \epsilon^{-at}$
16	$\frac{1}{s^2+a^2}$	$\frac{1}{a} \sin at$
17	$\frac{s}{s^2+a^2}$	$\cos at$
18	$\frac{1}{s^2-a^2}$	$\frac{1}{a} \sinh at$
19	$\frac{s}{s^2-a^2}$	$\cosh at$
20	$\frac{1}{s(s^2+a^2)}$	$\frac{1}{a^2} (1 - \cos at)$
21	$\frac{1}{s^2(s^2+a^2)}$	$\frac{1}{a^3} (at - \sin at)$
22	$\frac{1}{(s+a)(s^2+b^2)}$	$\frac{1}{a^2+b^2}$ $\left[\epsilon^{-at} + \frac{1}{b} \sqrt{a^2+b^2} \sin (bt - \theta) \right],$ $\theta = \tan^{-1} \left(\frac{b}{a} \right)$
23	$\frac{s}{(s+a)(s^2+b^2)}$	$-\frac{a}{a^2+b^2}$ $\left[\epsilon^{-at} - \frac{1}{a} \sqrt{a^2+b^2} \sin (bt + \theta) \right],$ $\theta = \tan^{-1} \left(\frac{a}{b} \right)$
24	$\frac{s^2}{(s+a)(s^2+b^2)}$	$\frac{a^2}{a^2+b^2}$ $\left[\epsilon^{-at} - \frac{b}{a^2} \sqrt{a^2+b^2} \sin (bt - \theta) \right],$ $\theta = \tan^{-1} \left(\frac{b}{a} \right)$
25	$\frac{1}{s[(s+a)^2+b^2]}$	$\frac{1}{a^2+b^2}$ $\left[1 - \frac{1}{b} \sqrt{a^2+b^2} \epsilon^{-at} \sin (bt + \theta) \right],$ $\theta = \tan^{-1} \left(\frac{b}{a} \right)$

26	$\frac{1}{s^2[(s+a)^2 + b^2]}$	$\frac{1}{a^2 + b^2}$ $\left[t - \frac{2a}{a^2 + b^2} + \frac{1}{b} e^{-at} \sin(bt + \theta) \right],$ $\theta = 2 \tan^{-1} \left(\frac{b}{a} \right)$
27	$\frac{s}{(s^2 + a^2)(s^2 + b^2)}$	$\frac{1}{b^2 - a^2} (\cos at - \cos bt)$
28	$\frac{1}{(s^2 + a^2)^2}$	$\frac{1}{2a^3} (\sin at - at \cos at)$
29	$\frac{s}{(s^2 + a^2)^2}$	$\frac{t}{2a} \sin at$
30	$\frac{s^2}{(s^2 + a^2)^2}$	$\frac{1}{2a} (\sin at + at \cos at)$
31	$\frac{s^2 - a^2}{(s^2 + a^2)^2}$	$t \cos at$
32	$\frac{1}{s(s^2 + a^2)^2}$	$\frac{1}{a^4} (1 - \cos at) - \frac{1}{2a^3} t \sin at$
33	$\frac{4a^3}{s^4 + 4a^4}$	$\sin at \cosh at - \cos at \sinh at$
34	$\frac{s}{s^4 + 4a^4}$	$\frac{1}{2a^2} \sin at \sinh at$
35	$\frac{1}{s^4 - a^4}$	$\frac{1}{2a^3} (\sinh at - \sin at)$
36	$\frac{s}{s^4 - a^4}$	$\frac{1}{2a^2} (\cosh at - \cos at)$
37	$\frac{s^2}{s^4 - a^4}$	$\frac{1}{2a} (\sinh at + \sin at)$
38	$\frac{s^3}{s^4 - a^4}$	$\frac{1}{2} (\cosh at + \cos at)$

Note 1: One can generate pairs in the table (or not in the Table) from some pairs in the Table. For example, we can generate Pair 10 from Pair 5 by taking the limit as parameter b approaches parameter a .

$$\frac{1}{(s+a)(s+b)} \leftrightarrow \frac{1}{b-a} (e^{-at} - e^{-bt})$$

$$\lim_{b \rightarrow a} \frac{1}{(s+a)(s+b)} = \frac{1}{(s+a)^2} \leftrightarrow \lim_{b \rightarrow a} \frac{1}{b-a} (e^{-at} - e^{-bt}) = \frac{0}{0}$$

Employ L'Hospital Rule:

$$\lim_{b \rightarrow a} \frac{1}{b-a} (e^{-at} - e^{-bt}) = \lim_{b \rightarrow a} \frac{\frac{d}{db} (e^{-at} - e^{-bt})}{\frac{d}{db} (b-a)} = \lim_{b \rightarrow a} t e^{-bt} = t e^{-at}$$

Resulting in Pair 10:

$$\frac{1}{(s+a)^2} \leftrightarrow t e^{-at}$$

Note 2: The transform of the rational function $\frac{s+a_0}{(s+a)^2}$ is not in the Table. We may rewrite this function as

$$\frac{s+a_0}{(s+a)^2} = \frac{s}{(s+a)^2} + \frac{a_0}{(s+a)^2}$$

and take advantage of the linearity and scaling of the s-operator along with Pairs 10 & 11 and write

$$\frac{s}{(s+a)^2} + \frac{a_0}{(s+a)^2} \leftrightarrow (1-at)e^{-at} + a_0 t e^{-at} = [(a_0 - a)t + 1]e^{-at}$$

Resulting in the new Pair

$$\frac{s+a_0}{(s+a)^2} \leftrightarrow [(a_0 - a)t + 1]e^{-at}$$

[All time-domain functions must be multiplied by the unit-step function $u(t)$]