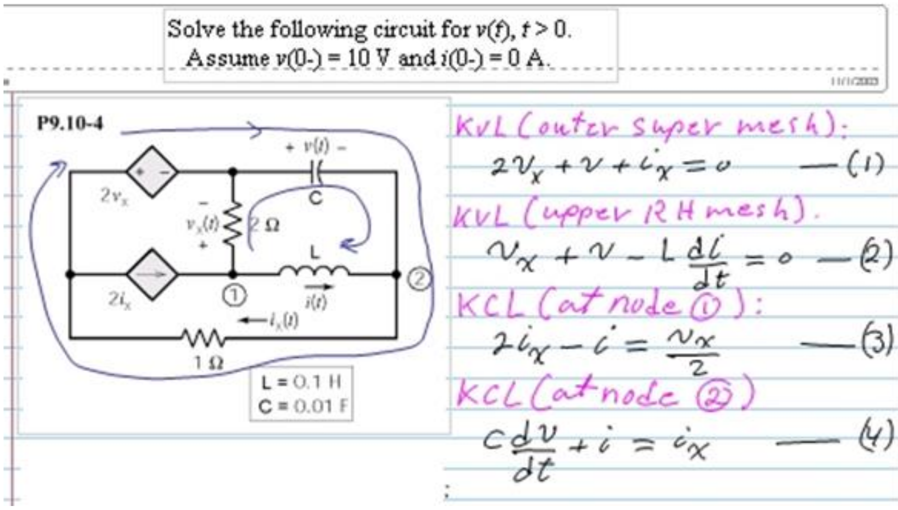


Given

$2Vx + V + Ix = 0$

Problem Using variable "I", so we use "I1"

$Vx + V - \frac{1}{10}\cdot(s\cdot I1) = 0$

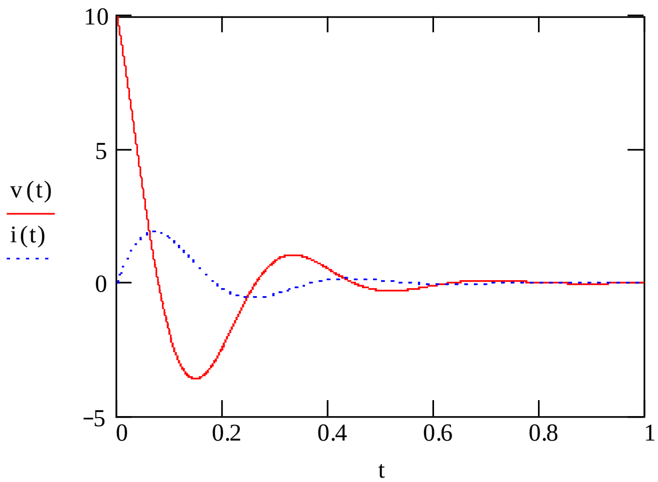
$2Ix - I1 = \frac{Vx}{2}$

$\frac{1}{100}\cdot(s\cdot V - 10) + I1 = Ix$

$$U(s) := \text{Find}(V, Vx, I1, Ix) \rightarrow \begin{bmatrix} \frac{10}{3}\cdot\frac{(20 + 9\cdot s)}{\left(40\cdot s + 1000 + 3\cdot s^2\right)} \\ \frac{-40}{3}\cdot\frac{(s+5)}{\left(40\cdot s + 1000 + 3\cdot s^2\right)} \\ \frac{500}{\left(120\cdot s + 3000 + 9\cdot s^2\right)} \\ \frac{-10}{3}\cdot\frac{(s-20)}{\left(40\cdot s + 1000 + 3\cdot s^2\right)} \end{bmatrix}$$

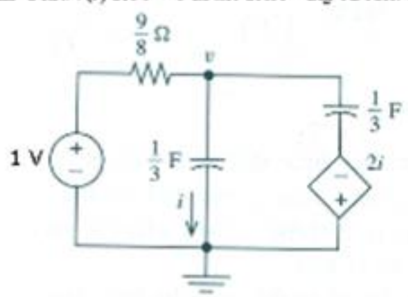
$$v(t) := U(s)_1 \text{ invlaplace}, s \rightarrow 10\cdot\exp\left(\frac{-20}{3}\cdot t\right)\cdot\cos\left(\frac{10}{3}\cdot\sqrt{26}\cdot t\right) - \frac{20}{39}\cdot\exp\left(\frac{-20}{3}\cdot t\right)\cdot\sqrt{26}\cdot\sin\left(\frac{10}{3}\cdot\sqrt{26}\cdot t\right)$$

$$i(t) := U(s)_3 \text{ invlaplace}, s \rightarrow \frac{25}{39}\cdot\exp\left(\frac{-20}{3}\cdot t\right)\cdot\sqrt{26}\cdot\sin\left(\frac{10}{3}\cdot\sqrt{26}\cdot t\right)$$



Example 2

III. Find $v(t)$ for $t > 0$ in the following circuit. Assume, $v(0+) = 0$ and $i(0-) = 0$.



Given

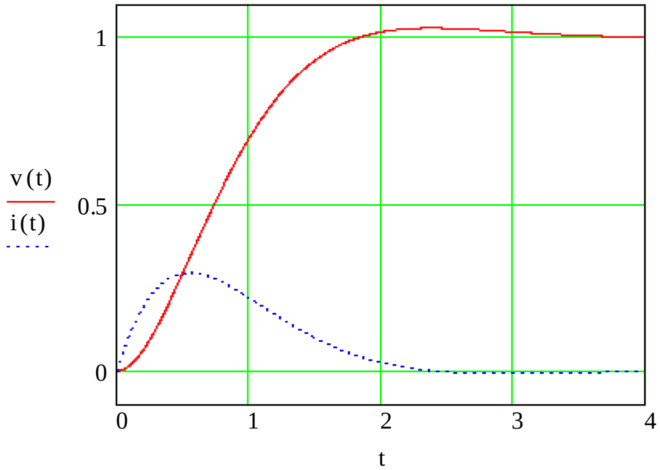
$$\frac{8}{9}\cdot\left(V-\frac{1}{s}\right)+I1+\frac{(V+2I1)\cdot s}{3}=0$$

$$I1=\frac{1}{3}\cdot s\cdot V$$

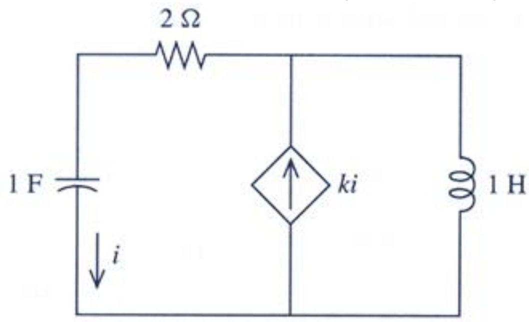
$$U(s):=\mathrm{Find}(V,I1)\rightarrow\begin{bmatrix}\dfrac{4}{s\cdot\left(4+3\cdot s+s^2\right)}\\ \dfrac{4}{\left(12+9\cdot s+3\cdot s^2\right)}\end{bmatrix}$$

$$v(t):=U(s)_1 \text{ invlaplace}, s\rightarrow 1-\exp\left(\frac{-3}{2}\cdot t\right)\cdot\cos\left(\frac{1}{2}\cdot\sqrt{7}\cdot t\right)-\frac{3}{7}\cdot\exp\left(\frac{-3}{2}\cdot t\right)\cdot\sqrt{7}\cdot\sin\left(\frac{1}{2}\cdot\sqrt{7}\cdot t\right)$$

$$i(t):=U(s)_2 \text{ invlaplace}, s\rightarrow\frac{8}{21}\cdot\exp\left(\frac{-3}{2}\cdot t\right)\cdot\sqrt{7}\cdot\sin\left(\frac{1}{2}\cdot\sqrt{7}\cdot t\right)$$



Example 3: Solve for i(t), t>0. Assume inductor current is zero at t=0 and initial capacitor voltage is 1V. Set k = -2.



Given

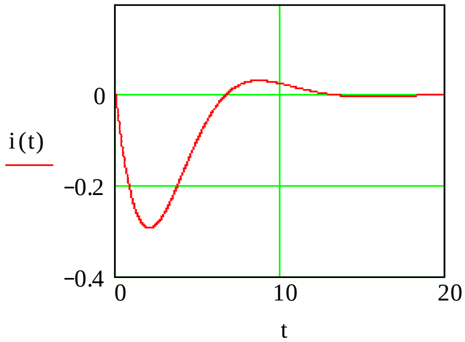
$s \cdot I2 - V - 2I1 = 0$

$s \cdot V - 1 = I1$

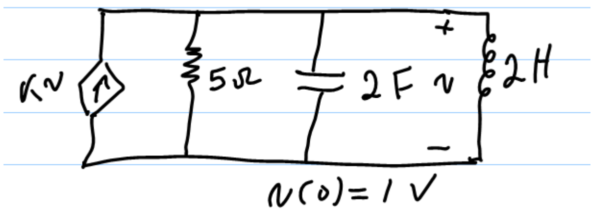
$I1 + I2 = -2 \cdot I1$

$$U(s) := \text{Find}(V, I1, I2) \rightarrow \begin{bmatrix} \dfrac{(3 \cdot s + 2)}{\left(3 \cdot s^2 + 2 \cdot s + 1\right)} \\ \dfrac{-1}{\left(3 \cdot s^2 + 2 \cdot s + 1\right)} \\ \dfrac{3}{\left(3 \cdot s^2 + 2 \cdot s + 1\right)} \end{bmatrix}$$

$$i(t) := U(s)_2 \text{ invlaplace}, s \rightarrow \frac{-1}{2} \cdot \exp\left(\frac{-1}{3} \cdot t\right) \cdot \sqrt{2} \cdot \sin\left(\frac{1}{3} \cdot \sqrt{2} \cdot t\right)$$



Example 8: Find k so that v(t) is a pure sinusoid (i.e., circuit oscillates).



Given

$$\frac{V}{5} + 2s \cdot V - 2 + \frac{V}{2s} = k \cdot V$$

$$V(s,k) := \text{Find}(V) \rightarrow \frac{-20}{\left(-2 \cdot s - 20 \cdot s^2 - 5 + 10 \cdot k \cdot s\right)} \cdot s$$

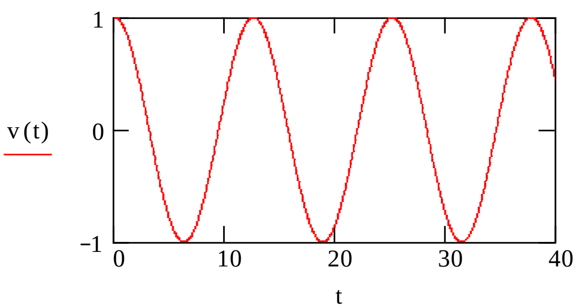
Circuit Natural frequencies (poles):

$$-2 \cdot s - 20 \cdot s^2 - 5 + 10 \cdot k \cdot s = 0 \text{ solve}, s \rightarrow \begin{bmatrix} \frac{-1}{20} + \frac{1}{4} \cdot k + \frac{1}{20} \cdot \left(-99 - 10 \cdot k + 25 \cdot k^2\right)^{\frac{1}{2}} \\ \frac{-1}{20} + \frac{1}{4} \cdot k - \frac{1}{20} \cdot \left(-99 - 10 \cdot k + 25 \cdot k^2\right)^{\frac{1}{2}} \end{bmatrix}$$

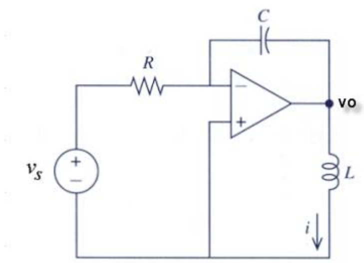
For circuit to oscillate, we need zero real part for the natural frequencies, or k=1/5:

$$v(t) := V\left(s, \frac{1}{5}\right) \text{ invlaplace}, s \rightarrow \cos\left(\frac{1}{2} \cdot t\right)$$

As expected for a paralle LC circuit, the oscillation frequency is 1/sqrt(LC)= 1/2



Example 9: Solve for i(t) and vo(t), t>0. Assume that at t=0, the capacitor voltage is 1 volt and i(0) = 1 amp. Let, vs(t)=sin(t)u(t), L=2H, C=1/5F and R=10 Ohm.



$$Vs(s) := \sin(t) \text{ laplace}, t \rightarrow \frac{1}{(s^2+1)}$$

Given

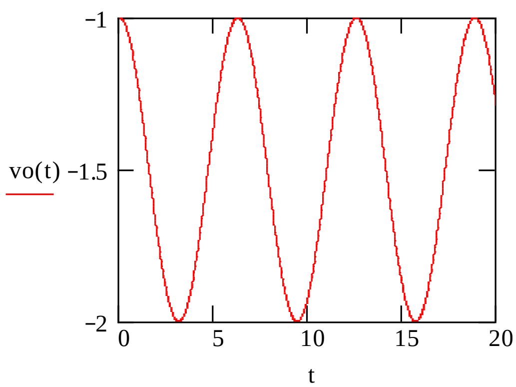
$$\frac{-s \cdot Vo - 1}{5} - \frac{Vs(s)}{10} = 0$$

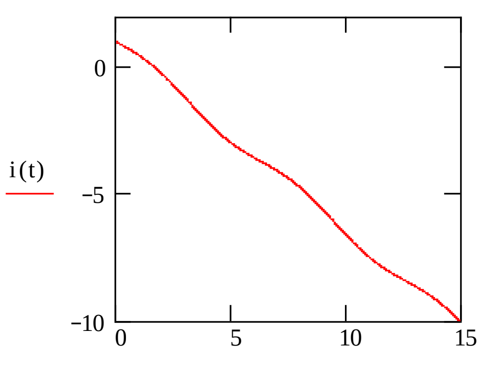
$$Vo = 2s \cdot I1 - 2$$

$$U(s) := \text{Find}(I1, Vo) \rightarrow \begin{bmatrix} \frac{1}{4} \cdot \frac{(4 \cdot s^3 + 4 \cdot s - 2 \cdot s^2 - 3)}{s^2 \cdot (s^2+1)} \\ \frac{-1}{2} \cdot \frac{(2 \cdot s^2 + 3)}{s \cdot (s^2+1)} \end{bmatrix}$$

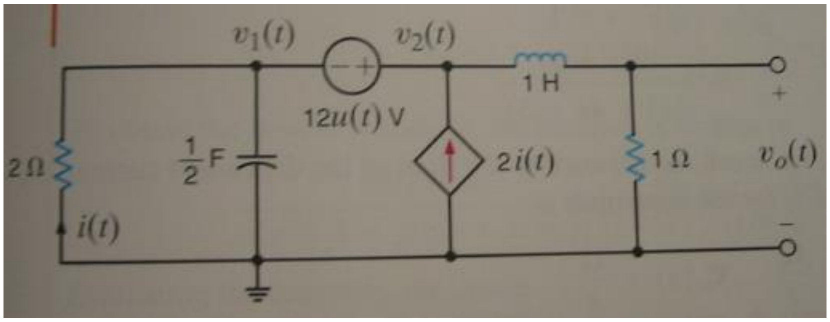
$$i(t) := U(s)_1 \text{ invlaplace}, s \rightarrow \frac{-3}{4} \cdot t + 1 + \frac{1}{4} \cdot \sin(t)$$

$$vo(t) := U(s)_2 \text{ invlaplace}, s \rightarrow \frac{-3}{2} + \frac{1}{2} \cdot \cos(t)$$

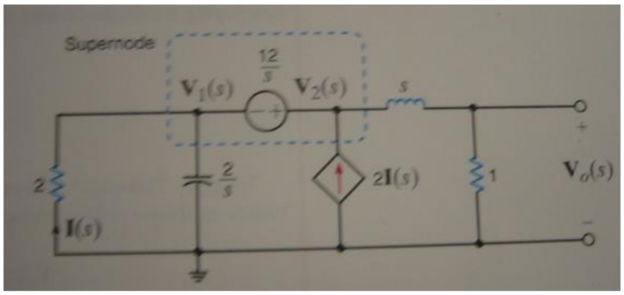




Example 10: Find vo(t), t>0. Assume zero initial energy stored in the circuit at t=0.



Transformed Circuit:



Given

$$\frac{V1}{2} + s\cdot\frac{V1}{2} - 2I1 + \frac{V2 - Vo}{s} = 0$$

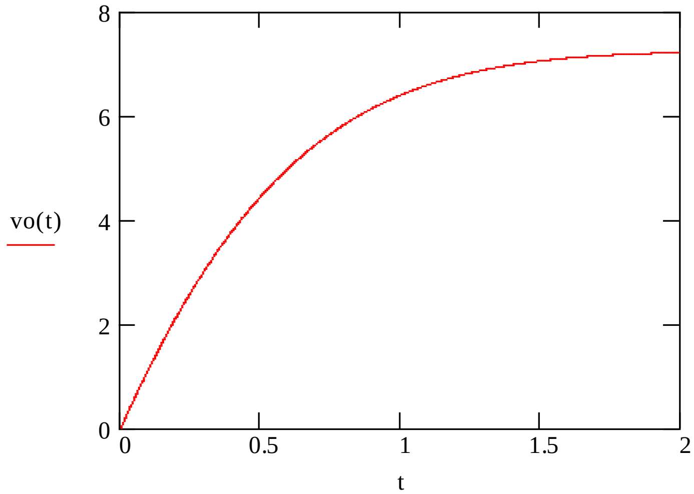
$$\frac{Vo - V2}{s} + Vo = 0$$

$$I1 = \frac{-V1}{2}$$

$$V2 - V1 = \frac{12}{s}$$

$$U(s) := \text{Find}(V1, V2, Vo, I1) \rightarrow \begin{bmatrix} \dfrac{-24}{s\cdot\left(5 + 4\cdot s + s^2\right)} \\ 12\cdot\dfrac{\left(3 + 4\cdot s + s^2\right)}{s\cdot\left(5 + 4\cdot s + s^2\right)} \\ 12\cdot\dfrac{(3 + s)}{s\cdot\left(5 + 4\cdot s + s^2\right)} \\ \dfrac{12}{s\cdot\left(5 + 4\cdot s + s^2\right)} \end{bmatrix}$$

$$vo(t) := U(s)_3 \text{ invlaplace}, s \rightarrow \frac{36}{5} - \frac{36}{5}\cdot\exp(-2\cdot t)\cdot\cos(t) - \frac{12}{5}\cdot\exp(-2\cdot t)\cdot\sin(t)$$


vo(t)
8
6
4
2
0
0
0.5
1
1.5
2
t