

THE BRAIN

I. INTRODUCTION

A. Human Brain

- 1) mass $\sim 1 - 2$ kg in mature adult
 - a) about 2% of body weight
 - i) uses 20% of oxygen, 25% of glucose, 15% of blood flow
 - b) mass at birth about 20% of final value
 - i) mass increase due to growth of axons, dendrites, synapses, myelin sheaths

B. Cortex

- 1) size of cortex separates humans from other species
 - a) area: 5 cm^2 for rat, $5 \times 10^2 \text{ cm}^2$ for chimp, $2 \times 10^3 \text{ cm}^2$ for human
 - i) extra area in human cortex obtained by folding
 - ii) thickness of cortex $\sim 0.3 \text{ cm}$
 - b) $\gtrsim 3 \times 10^{10}$ neurons in human cortex
 - i) mammalian cortex has $\sim 1.5 \times 10^7$ neurons per cm^{-2}
 - c) $\gtrsim 10^{14}$ synapses in human cortex
 - i) $\gtrsim 10^3$ synapses per neuron
- 2) human genome does not carry detailed wiring diagram for cortex
 - a) its information content is far too small
 - i) wiring diagram would require $\gtrsim 10^{14}$ bits of information
 - b) genome carries about 5×10^9 bits of information
 - i) human genome is about one meter of DNA
 - ii) 4 types of base pairs
 - iii) separation of $4 \times 10^{-8} \text{ cm}$ between base pairs
 - iv) much of genome may be nonsense
- 3) cortex develops in response to external stimuli
 - a) molecular markers involved in initial wiring
 - b) refinements due to activity
 - c) number of synapses pared back during development

C. Explosion In Size Of Cortex Due To Limited Genetic Instructions

- 1) comparison of evolution of genome and cortex
 - a) genome length: 4 cm for fruit fly, 40 cm for chicken, 1 m for mouse, 1 m for human

- b) number of neurons: 10^5 for fruit fly, 5×10^6 for mouse, 10^{11} for human
- 2) rapid evolution during past 3×10^6 yr

II. FUNCTION

A. Neuron

- 1) components: cell body, axon, dendrites
- 2) nominal dimensions for pyramidal cell in cortex
 - a) cell body: blob with $r \sim 20 \mu\text{m}$
 - b) axon: cylinder with $r \sim 1 \mu\text{m}$ and $l \sim 1 \text{cm}$
 - c) total surface area about $6 \times 10^{-4} \text{cm}^2$
 - i) dominated by axon
 - d) total volume about 10^{-8}cm^3
 - i) comparable contributions from axon and cell body
- 2) electrical properties
 - a) axons are output devices
 - i) actively propagate signals
 - ii) contain repeater stations
 - b) dendrites are input devices
 - i) electrically passive
 - ii) some may produce spikes
- 3) synapses
 - a) connect axons to dendrites
 - i) signals transmitted chemically across synapses
 - ii) synaptic space $\approx 2 \times 10^{-6} \text{cm}$
 - iii) time delay $\sim 0.1 \text{ms}$ due to diffusion
 - iv) to achieve post synaptic threshold may take much longer
 - b) can be excitatory or inhibitory
 - i) excitatory: glutamate transmitter opens Na channels (MSG)
 - ii) inhibitory: gaba transmitter opens Cl or K channels
 - iii) most common neurotransmitters in cortex
 - iv) amino acids
 - v) inhibitory synapses usually more proximal to cell body

B. Action Potentials

- 1) neuron sums inputs

- a) strength related to distance of synapse on dendrite from cell body
- b) sum determines whether firing occurs
- 2) spikes initiated on axon close to cell body
 - a) can travel in both directions along axon
- 3) pulses are quantized, all the same
 - a) pulse length ~ 1 ms
 - b) pulse strength $\Delta V \sim 10^2$ mV
 - c) length and strength determined by kinetics of ion channels
- 4) signal strength coded in firing rate ν
 - a) at rest: $0 \lesssim \nu \lesssim 50$ Hz, typically $\nu \sim 5$ Hz
 - b) excited: $2 \lesssim \nu \lesssim 200$ Hz, typically $\nu \sim 50$ Hz
 - i) limited to $\nu \lesssim 10^3$ Hz by refractory period of ion channels
- 5) propagation speed
 - a) depends on axon diameter and myelination
 - i) $v \sim 5$ m s $^{-1}$ typical value for brain
 - ii) up to $v \sim 100$ m s $^{-1}$ in spinal cord

C. Axon Modeled As Coaxial Cable

- 1) parameters
 - a) radius a , membrane thickness t , length, l
 - i) typical values: $a \sim \mu\text{m}$, $t \sim 7 \times 10^{-7}$ cm, $l \sim 1$ cm
 - b) longitudinal resistance, $R_a = \rho_a / \pi a^2 l$
 - i) salt solution, $\rho_a \approx 30$ ohm cm
 - ii) $R_a \sim 1 \times 10^9 (\mu\text{m}/a)^2 (l/\text{cm})$ ohm
 - c) membrane capacitance, $C_m = 2\pi\epsilon_0 K a l / t$
 - i) dielectric constant of lipid membrane, $K \approx 6$
 - ii) $C_m \approx 7 \times 10^{-4} (a/\mu\text{m})(l/\text{cm}) \mu\text{F}$
 - iii) $C_m/A \approx 1 \mu\text{F cm}^{-2}$
 - d) membrane resistance, $R_m = \rho_m t / (2\pi a l)$
 - i) $\rho_m \approx 1.5 \times 10^9$ ohm cm
 - ii) $\rho_m t \approx 10^3$ ohm cm 2
 - iii) $R_m \approx 2 \times 10^6 (\mu\text{m}/a)(\text{cm}/l)$ ohm
 - e) inductance negligible

D. Action Potential Propagation Along Unmyelinated Neuron

- 1) longitudinal diffusion

a) neglect current through membrane

$$\frac{\partial V}{\partial t} = -\frac{l}{C_m} \frac{\partial I}{\partial z}$$

$$\frac{\partial V}{\partial z} = -\frac{R_a}{l} I$$

b) diffusion equation

$$\frac{\partial V}{\partial t} = \frac{l^2}{R_a C_m} \frac{\partial^2 V}{\partial z^2}$$

ii) diffusion constant, $D \equiv l^2/R_a C_m \sim 1.5(a/\mu\text{m}) \text{ cm}^2 \text{ s}^{-1}$

c) $\lambda \sim (D\Delta t)^{1/2}$, spreading length for pulse of duration Δt

i) $\lambda \sim 4 \times 10^{-2}(a/\text{cm})^{1/2}(\Delta t/\text{ms})^{1/2} \text{ cm}$

d) propagation speed along axon

$$v \sim \frac{\lambda}{\Delta t} \sim 40 \left(\frac{a}{\mu\text{m}} \right)^{1/2} \text{ cm s}^{-1}$$

i) evaluated for $\Delta t \approx 1 \text{ ms}$

2) leakage through membrane

a) clamp voltage of axoplasm

$$\frac{\partial V}{\partial t} = -\frac{V}{R_m C_m}$$

b) voltage decays exponentially with time constant $\tau = R_m C_m$

i) $\tau \sim 1.5 \times 10^{-3} \text{ s}$

3) combined equation reads

$$\frac{\partial V}{\partial t} + \frac{V}{R_m C_m} = \frac{l^2}{R_a C_m} \frac{\partial^2 V}{\partial z^2}$$

a) impulse regeneration not included in equation

E. Action Potential Propagation Along Myelinated Neuron

1) myelin sheath decreases C_m

a) 10 – 15 wraps of myelin sheath per micron diameter of axon

i) like paper towels on cardboard roller

b) $C_m \sim 3 \times 10^{-5}(l/\text{cm}) \mu\text{F}$

i) note C_m independent of a

- 2) cross membrane currents restricted to nodes of Ranvier
 - a) separated by a few mm
 - a) size a few μm
- 3) myelination increases propagation speed at fixed size
 - a) $\lambda \sim 2 \times 10^{-1}(a/\mu\text{m}) \text{ cm}$
 - b) $v \sim 2(a/\mu\text{m}) \text{ m s}^{-1}$
 - i) note: $v \propto a$

F. Power Requirements

- 1) $P = C(\Delta V)^2 \nu / 2$
 - a) $P \sim 3 \times 10^{-12} \nu \text{ watt}$ for our canonical neuron
 - b) $\sim 10^{11}$ neurons firing at $\nu \sim 10 \text{ Hz}$ yields a total power $\sim 3 \text{ watt}$
 - c) based on unmyelinated axons
 - i) myelination decreases power usage
 - ii) do small, unmyelinated axons use most of electric power?
 - iii) could dendrites use significant power?
- 2) total power used by brain $\sim 20 \text{ watt}$
 - a) how much for ion pumps?
 - b) how much for axon transport?
 - i) molecular motors
- 3) experimental indications
 - a) ion transport is major part of metabolism
 - i) barbiturate anesthesia producing isoelectric EEG reduces metabolism to 40% of normal value
 - ii) inhibiting Na-K pump using ouabain reduces metabolism to 20% of normal value

III. INFORMATION INPUT AND STORAGE

A. Eye

- 1) retina is 2.4 cm behind cornea, pupil size $0.2 \lesssim p \lesssim 0.4 \text{ cm}$
 - a) 10^7 cones
 - i) maximum density in center of fovea, $1.5 \times 10^7 \text{ cm}^{-2}$
 - b) 10^8 rods
 - i) maximum density 20° from center of fovea, $1.6 \times 10^7 \text{ cm}^{-2}$
 - ii) can detect single photon
 - c) cone acuity 10 times rod acuity, less convergence

- d) cone sensitivity at fovea 10 times smaller than rod sensitivity at 20°
- 2) resolution of eye at fovea $\Delta\theta \sim 5 \times 10^{-4} \text{ rad} \sim 2 \text{ arc minutes}$
 - a) density of cones matches diffraction limited resolution of eye
 - i) diffraction limit: $\Delta\theta \sim \lambda/p \sim 2.5 \times 10^{-4}$
 - ii) cone spacing: $\Delta\theta \sim 10^{-4}$
- 3) input from visual receptors funnels into $\sim 10^6$ neurons in optic nerve
 - a) optic nerve can transmit $\sim 10^7$ bits per second
- 4) auditory nerve has $\sim 3 \times 10^4$ neurons
 - a) auditory bandwidth is $\sim 2 \times 10^4 \text{ Hz}$

B. Television

- 1) standard TV channel uses $\Delta\nu \approx 6 \text{ MHz}$ in range $50 - 1,000 \text{ MHz}$
 - a) only $\Delta\nu \approx 4 \text{ MHz}$ for picture
 - b) 2.11×10^5 picture elements
 - i) 495 horizontal lines
 - c) raster scans at 60 frames per second
 - i) more than 40 frames per second needed to avoid flicker
 - d) 10^7 elements per second
- 2) angular scale of picture element
 - a) $50 \text{ cm} \times 50 \text{ cm}$ screen
 - i) element size, $\Delta x \approx \Delta y \approx 0.1 \text{ cm}$
 - b) viewed at distance of $d \approx 3 \text{ m}$
 - c) $\Delta\theta \sim 3 \times 10^{-4} \text{ rad}$
- 3) match of visual input to TV
 - a) $\Delta\nu \approx 4 \text{ MHz}$ matches capacity of optic nerve
 - i) $\sim 10^6$ neurons firing at $\nu \sim 10 \text{ Hz}$
 - b) $\Delta\theta \sim 5 \times 10^{-4} \text{ rad}$ matches angular separation of picture elements
- 4) aliens might wonder which came first, the TV or the eye

C. Memory

- 1) Hebb proposed that information is stored in strength of synaptic connections
 - a) suppose that there are there N discernible levels of synaptic strength
 - b) human brain might be able to store $\gtrsim 10^{14} \ln_2(N)$ bits
 - i) every bit in one full year of viewing TV