ECE 512 – Winter 1998
Test #2
WSU - ECE
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I. Consider the following function $F(x) = \dfrac{1}{x^3 - \frac{3}{4}x - \frac{1}{2}}$.

a. Find the second order Taylor series approximation for $F(x)$ at the point $x = -0.5$.
b. Compute the approximation error at $x = -0.4$ between $F(x)$ and its approximation in part a. Repeat for $x = 0$.

II. Consider the quadratic function $F(x_1, x_2) = 5x_1^2 - 6x_1x_2 + 5x_2^2 + 4x_1 + 4x_2 + 5$.

a. Express $F(\mathbf{x})$ as a standard quadratic form; i.e., $F(\mathbf{x}) = \frac{1}{2}\mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x} + \mathbf{d}^{\mathrm{T}}\mathbf{x} + c$.
b. Find the stationary point $\mathbf{x}^*$.
c. Is the stationary point a minimum, maximum, or saddle point.
d. (*optional*) Find the eigenvectors at the stationary point and provide a rough sketch of the contour plot. (*5 bonus points*)
e. (*optional*) Generate a rough sketch of the search trajectory employing exact steepest gradient descent search. Assume that the search starts at the origin. (*5 bonus points*)

III. Consider the following unlabelled training set:

$$\mathbf{x}^1 = \begin{bmatrix} .5 \\ .5 \end{bmatrix}; \ \mathbf{x}^2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \ \mathbf{x}^3 = \begin{bmatrix} -.5 \\ .5 \end{bmatrix}; \ \mathbf{x}^4 = \begin{bmatrix} -.5 \\ -.5 \end{bmatrix}; \ \mathbf{x}^5 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}; \ \mathbf{x}^6 = \begin{bmatrix} .5 \\ -.5 \end{bmatrix}$$

Employ simple competitive learning as described by Equations (3.4.2) and (3.4.5) in your textbook to find the weight vectors in a two-unit competitive net after one complete cycle over the training set. Assume $\rho = 1$ and an ordered presentation of the input vectors (i.e., $\mathbf{x}^1$, $\mathbf{x}^2$, $\mathbf{x}^3$, …, $\mathbf{x}^6$). Also assume initial weights $\mathbf{w}_1 = [0 \ 1]^{\mathrm{T}}$ and $\mathbf{w}_2 = [-.5 \ -.5]^{\mathrm{T}}$.

IV. Solve Problem 3.4.1 on page 140 of your textbook. (*Correction:* $\sum_{j=1}^{n} w_{ij}^k = 1$ for all k.)