ECE 512 – Winter 1998
Final Exam
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I. Consider the following function $y(x) = (x^2 + 2x)\cos x$.

a. Find the second order Taylor series approximation (referred to as $\hat{y}$) for $y(x)$ at the point $x^* = 3.5725$.
b. Employ one iteration of Newton's search method (i.e., $x^{new} = x^c - \rho \left.\frac{dg}{dx}\right|_{x^c}$, $where\ \rho = \left(\left.\frac{d^2 g}{dx^2}\right|_{x^c} \right)^{-1}$) on $y(x)$ (i.e., set $g = y$) to find x^{new} starting from $x^c = 2.5$.
c. Repeat part b using the approximation function in part a (i.e., set $g = \hat{y}$).
d. Compute the error $\left| x^* - x^{new} \right|$ in parts b and c. Discuss your results.

II. Consider a single unit with the transfer function $y = \tanh(\beta\, net)$, where $net = \sum_{i=1}^{n} w_i x_i$. Derive a learning rule for w_i to learn associations of the form ($\mathbf{x}$, d) employing steepest gradient descent for minimizing the criterion function:

$$E(\mathbf{w}) = \left[(1+d)\ln\left(\frac{1+d}{1+y}\right) + (1-d)\ln\left(\frac{1-d}{1-y}\right) \right]$$

III. Consider the following training set ($\mathbf{x}^1$, 1) and ($\mathbf{x}^2$, -1) where

$$\mathbf{x}^1 = [1\ 1]^T \text{ and } \mathbf{x}^2 = [1\ -1]^T.$$

a. Find the output unit's optimal weight vector $\mathbf{w}^* = [w_1\ w_2]^T$ in a two hidden unit radial basis function network (assume a linear output unit and employ the "exact " interpolation method). Assume hidden unit j (j = 1, 2) to have Gaussian transfer function of the form:

$$z_j(\mathbf{x}) = \exp\left(-\frac{\left\| \mathbf{x} - \boldsymbol{\mu}_j \right\|^2}{4} \right)$$

b. Compute the outputs of the above net for the inputs $\mathbf{x}^1$ and $\mathbf{x}^2$, respectively.
c. How would adding a bias input ($z_0 = 1$) with an associated non-zero weight w_0 at the output unit affect the results in part *b*, assuming the solution found in part *a* is used?