

Introduction to **Electric Circuits**

The following slides were designed by:

Professor Mohamad Hassoun

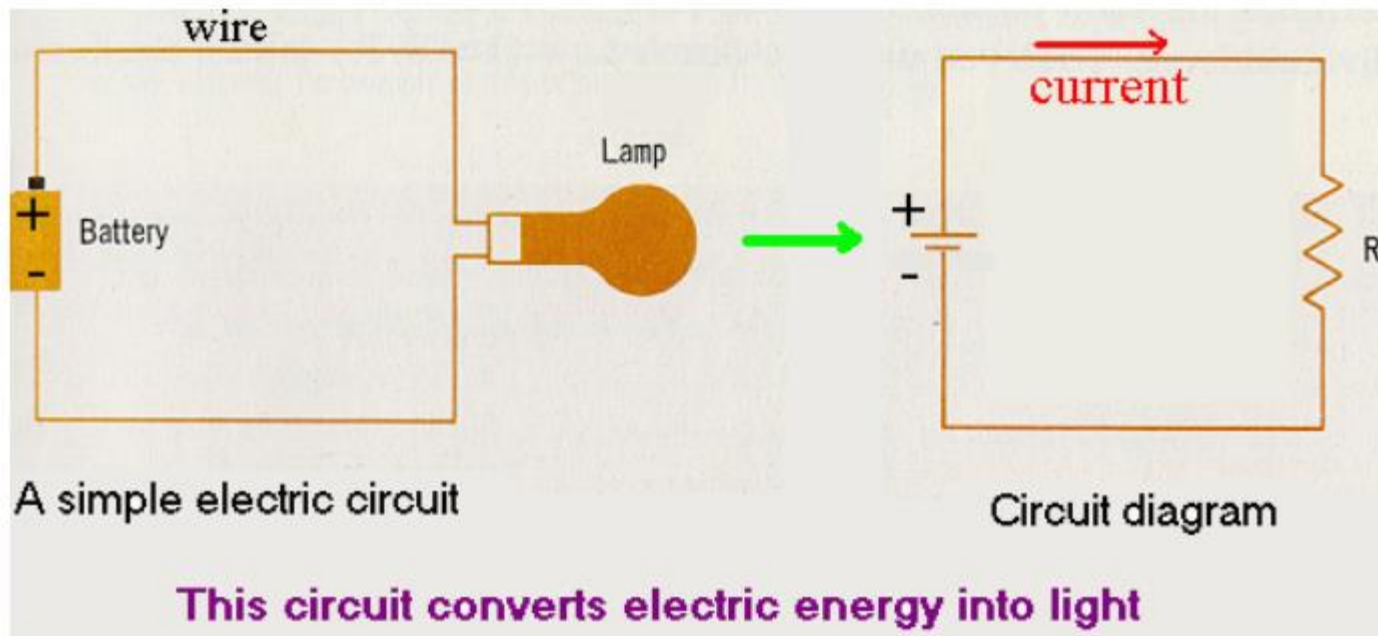
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Preliminary Definitions

Electricity is the physical phenomenon arising from the existence and interaction of electric charge. An **electric circuit** is an interconnection of electrical elements linked together in a closed path so that an electric current may flow continuously in order to manipulate electric energy.



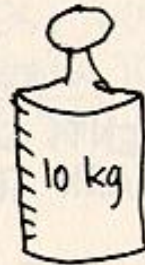
Preliminary Definitions (cont.)

Engineering combines the study of mathematics and natural and social sciences to direct the forces of nature for the benefit of humankind.

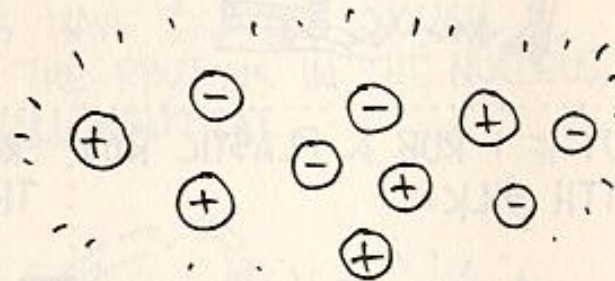
Electrical Engineering deals with the design and production of devices (often called hardware) based on electric circuits.

The outstanding characteristics of electricity when compared with other energy sources are its mobility and flexibility. Electrical energy can be moved to any point along a couple of wires and can be converted to light, heat or mechanical force.

IN MECHANICS WE USED THE BASIC PROPERTY OF MATTER CALLED **MASS**. IN ELECTRICITY, THE BASIC CONCEPT IS **CHARGE**.



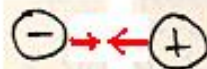
MECHANICAL
CONCEPT



ELECTRICAL CONCEPT



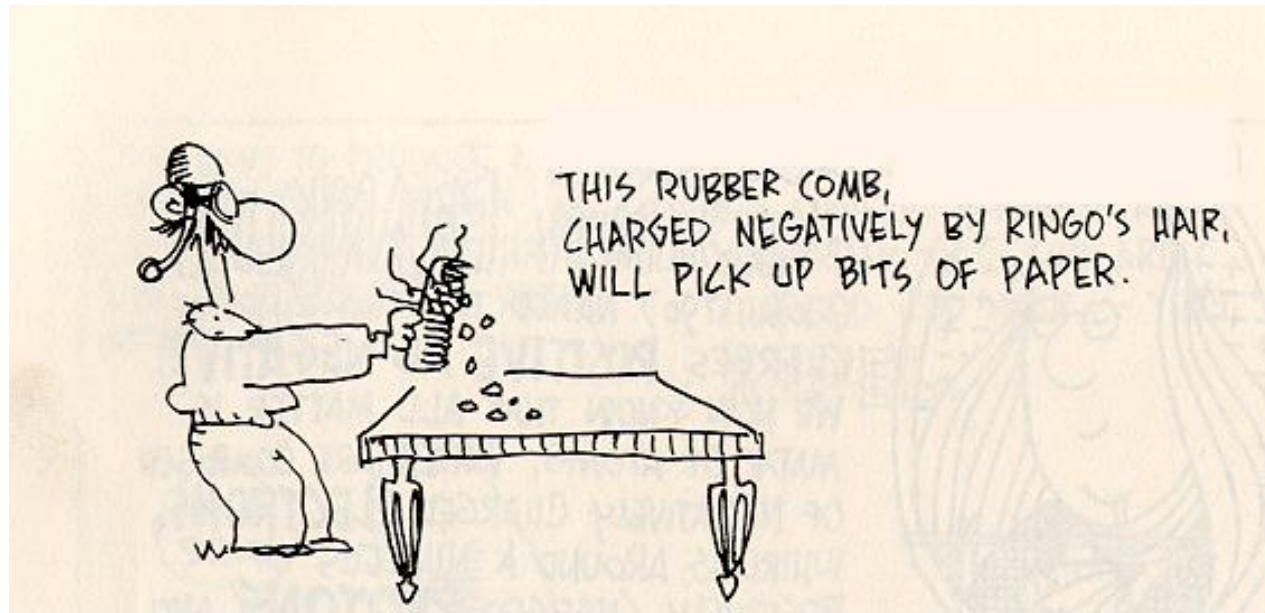
In the late 1740s, Benjamin Franklin developed the theory that there are two kinds of charge, positive and negative.



Unlike charges
attract



Like charges
repel



The philosopher Thales (640-546 B.C.) is thought to have been the first person who observed the electrical properties of amber.

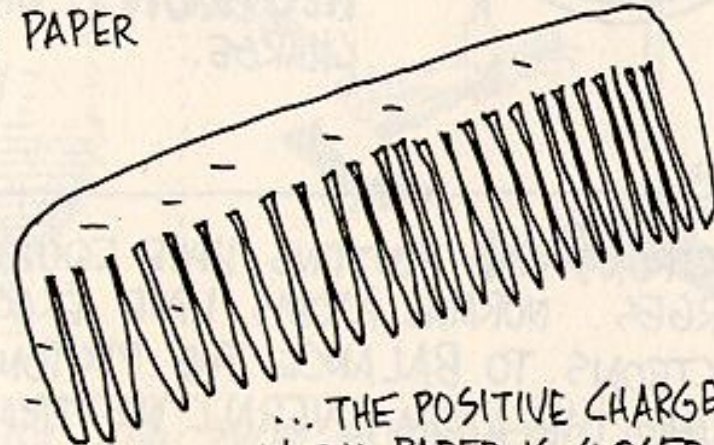
He noted that when amber was rubbed, it acquired the ability to pick up light objects such as straw and dry grass.

IT DOES SO BECAUSE THE PAPER
BECOMES ELECTRICALLY

POLARIZED:

THE NEGATIVE COMB
REPELS ELECTRONS
IN THE PAPER AND
ATTRACTS THE POSITIVE
NUCLEI OF THE ATOMS
IN THE PAPER.

THERE IS A CHARGE
SHIFT IN THE
PAPER! EVEN
THOUGH IT IS
NEUTRAL OVERALL ...

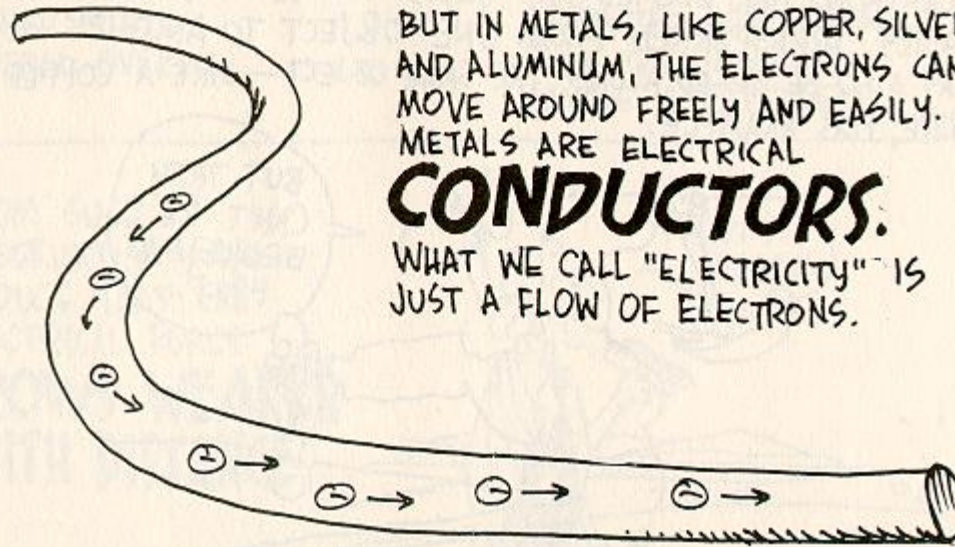
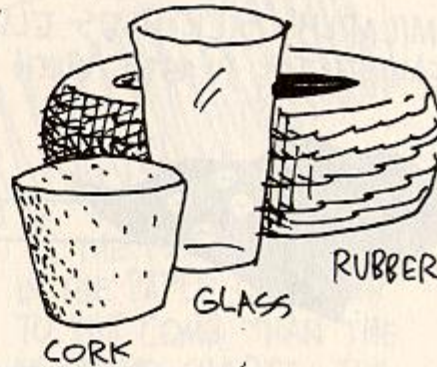


... THE POSITIVE CHARGE
IN THE PAPER IS CLOSER
TO THE COMB THAN THE
NEGATIVE CHARGE. THE
POSITIVE CHARGE IS THEN
ATTRACTED MORE STRONGLY
THAN THE NEGATIVE
CHARGE IS REPELLED!

MATERIALS LIKE RUBBER, GLASS,
AND PLASTIC ARE ELECTRICAL

INSULATORS:

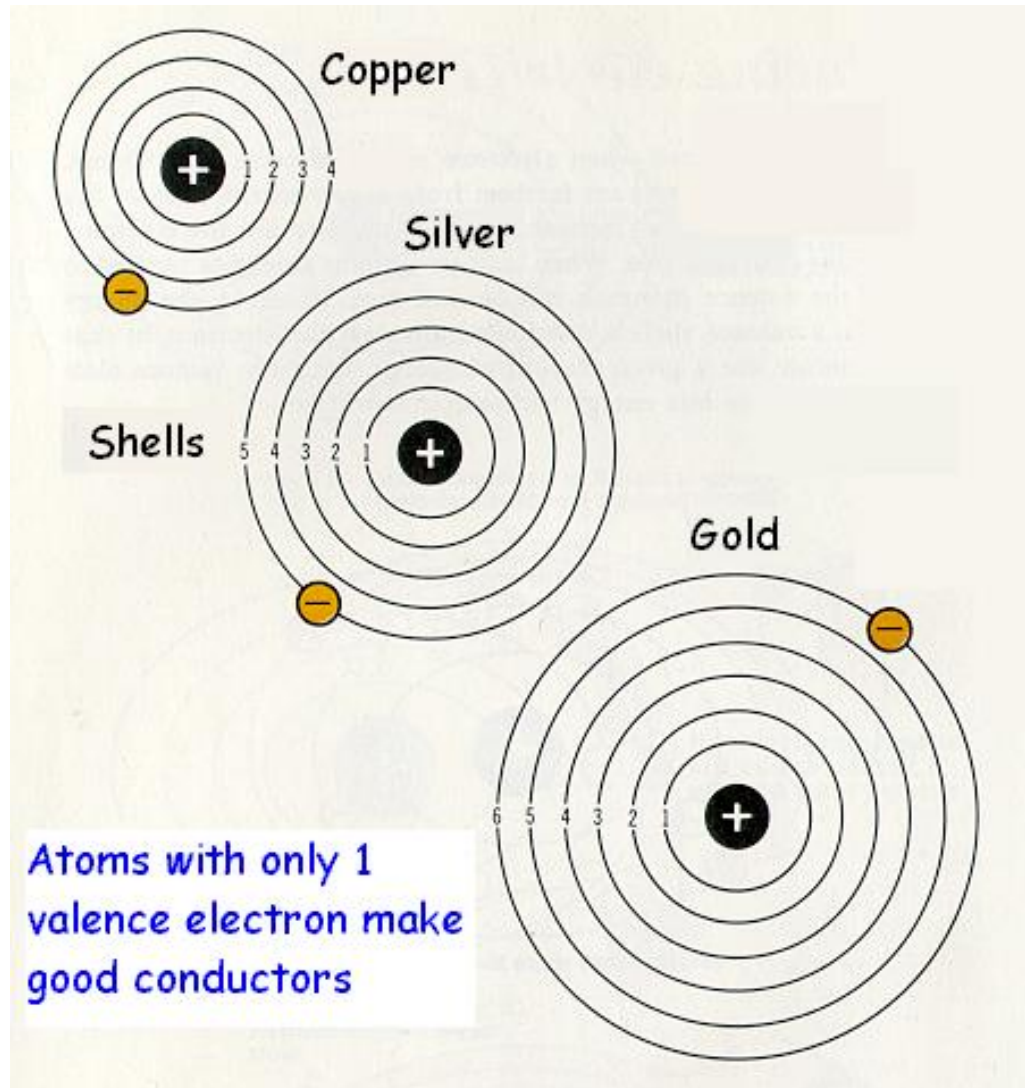
CHARGE CAN BE RUBBED ON OR
OFF THEIR SURFACES, BUT IT
TENDS TO STICK THERE AND
WILL NOT MOVE EASILY
THROUGH THE MATERIALS.

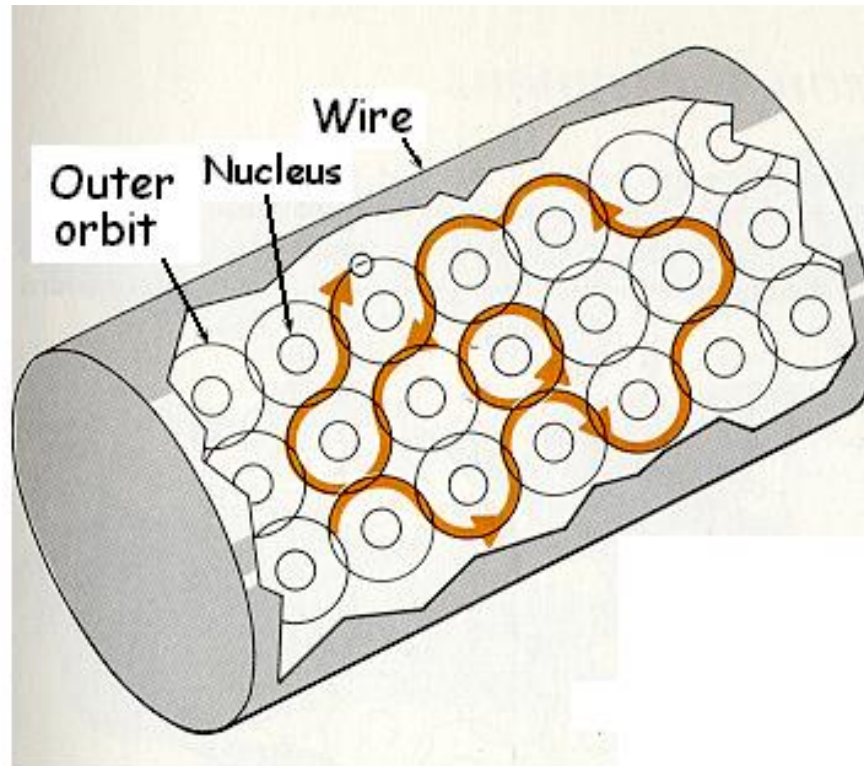


BUT IN METALS, LIKE COPPER, SILVER,
AND ALUMINUM, THE ELECTRONS CAN
MOVE AROUND FREELY AND EASILY.
METALS ARE ELECTRICAL
CONDUCTORS.
WHAT WE CALL "ELECTRICITY" IS
JUST A FLOW OF ELECTRONS.

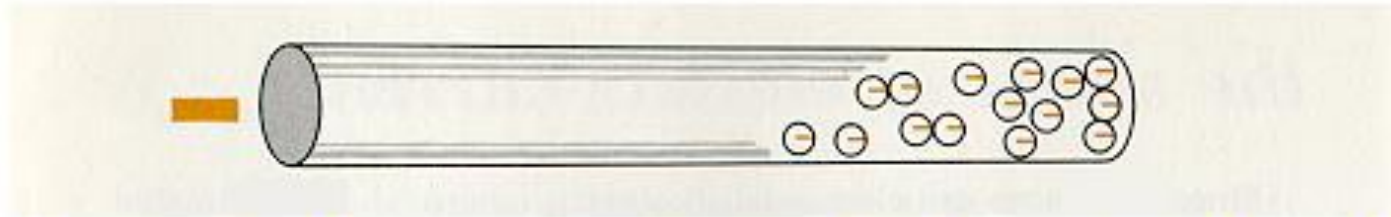
Charge carriers in semiconductors (transistors, diodes):

There are two types of charge carriers in semiconductors. One is electrons (just like conductors) and the other are the traveling vacancies in the valence band (holes) which carry a positive charge equal in magnitude to that of an electron. You will learn more about this in your electronics courses.



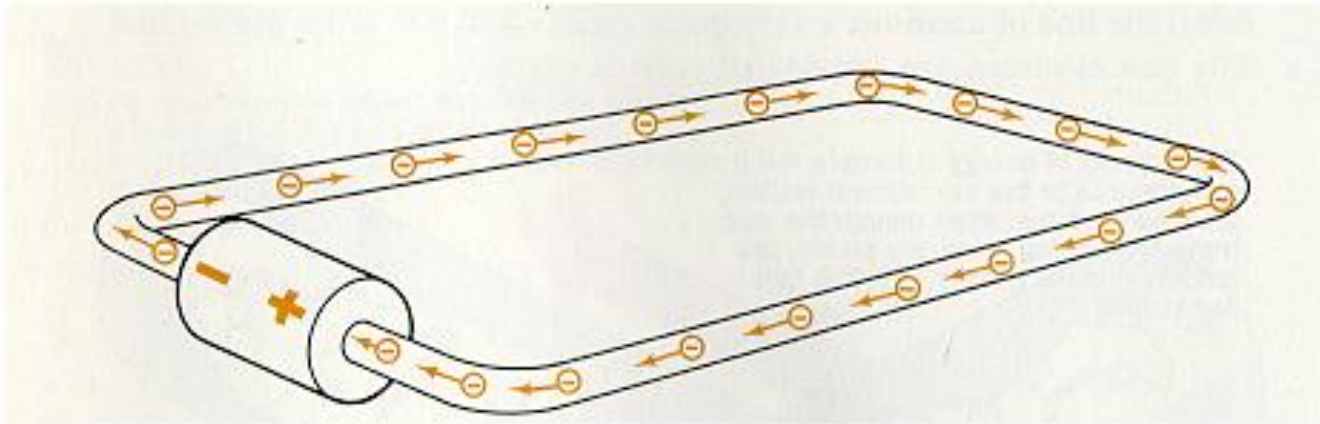


The free electrons in a copper wire change orbits in a random manner. All atoms share their outer electrons and become bound together. This is called metallic bonding

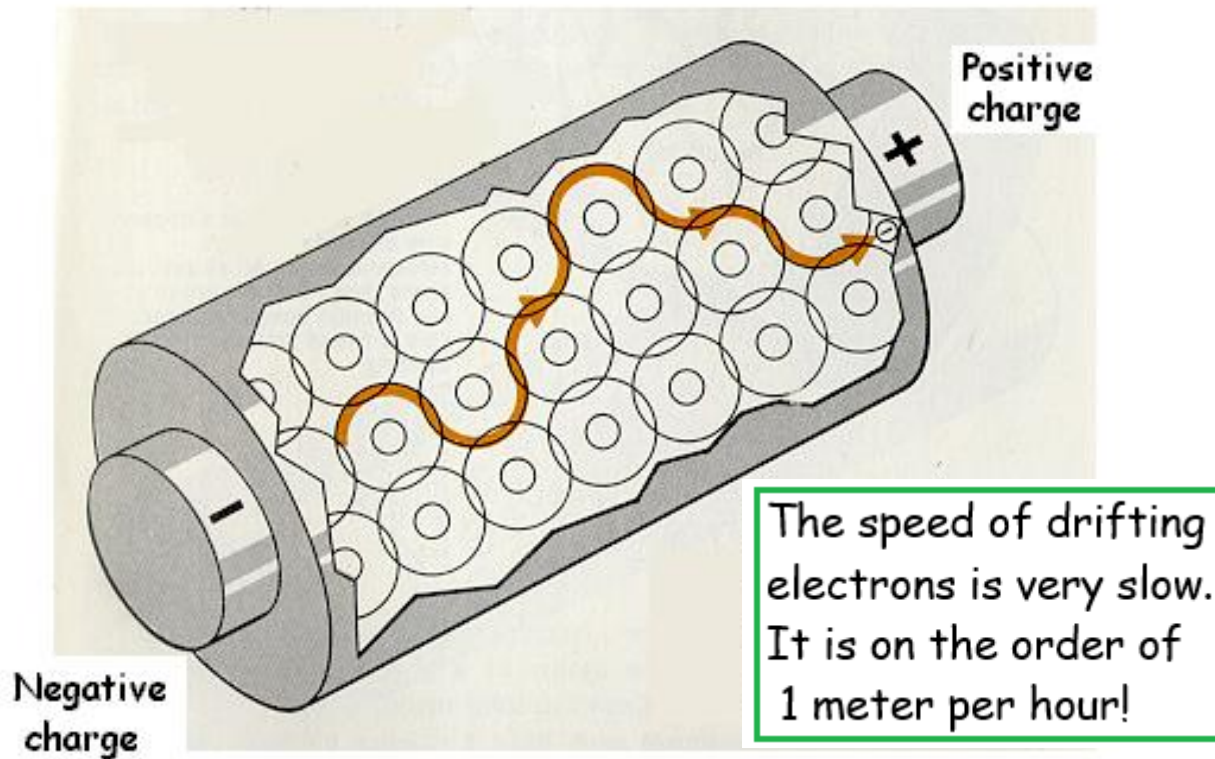


A negative charge put at one end of a wire will repel electrons to the other end until an equal charge is built up to stop electrons from flowing.

A complete or closed circuit



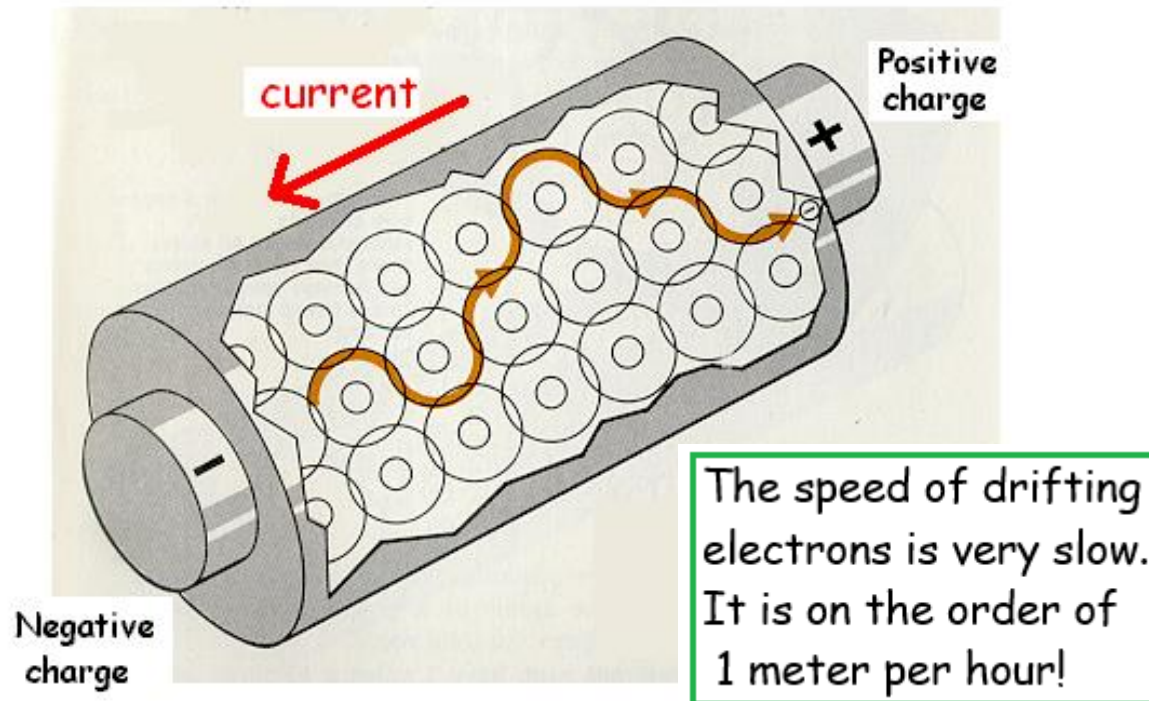
But, if a power source (e.g., battery) applies opposite charges to each end of the wire, electrons will continue to flow. An electron leaves the negative side of the battery for each electron that enters the positive side.



The free electron is repelled by the negative charge and is attracted by the positive charge. It changes orbit and **drifts** toward the positive charge.

This free electron movement in response to an external power source produces **electric current**.

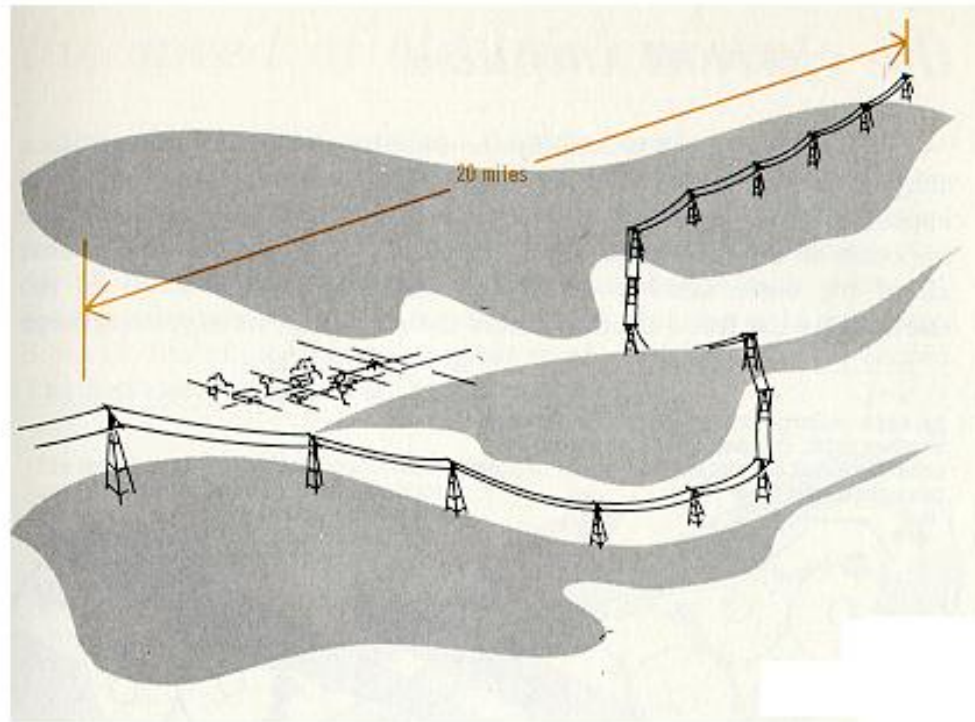
Electric Current



Electric current is defined in terms of two quantities: **Direction and magnitude.**

But first what is the speed of electric current?

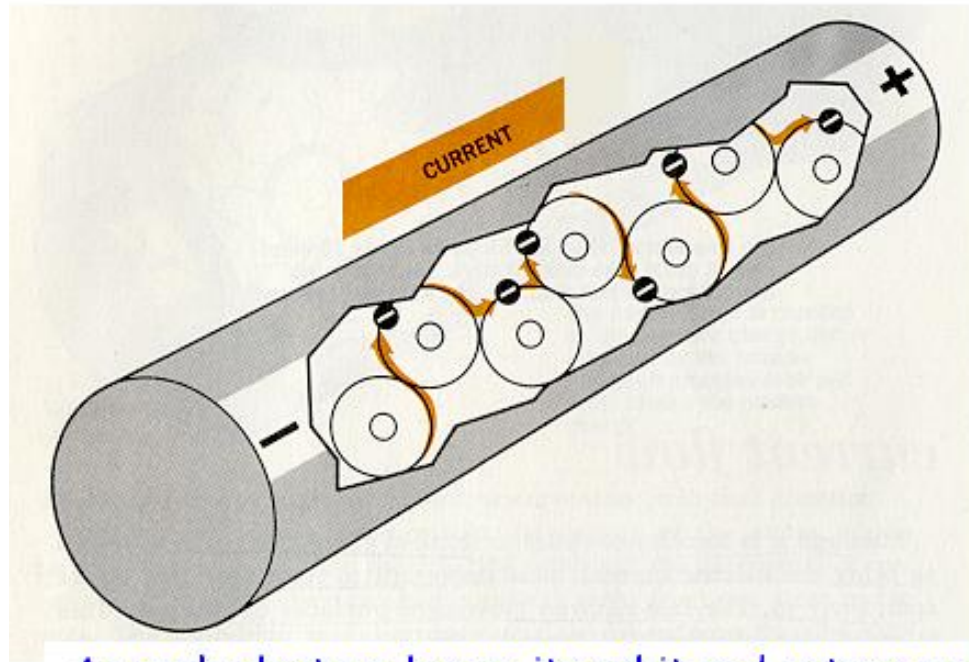
Speed of Electric Current



If a free electron had to drift down a wire 20 miles long, it could take over 30 days. Yet an electric current travels that distance in a fraction of a second.

Charge carriers in the wires of electric circuits are electrons. These electrons are simply supplied by the atoms of copper (or whatever material the wire is made of) within the metal wire. Once the switch is turned to *on*, the circuit is closed and there is an electric potential difference is established across the two ends of the external circuit. The electric field signal travels at nearly the speed of light to all mobile electrons within the circuit, ordering them to begin *marching*. As the signal is received, the electrons begin moving along a zigzag path in their usual direction. Thus, the flipping of the switch causes an immediate response throughout every part of the circuit, setting charge carriers everywhere in motion in the same net direction. While the actual motion of charge carriers occurs with a slow speed, the signal that *informs* them to start moving travels at a fraction of the speed of light.

Electric current speed

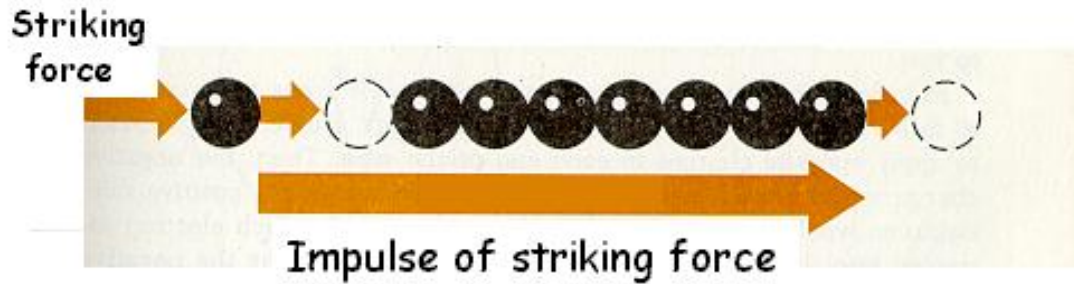


As each electron leaves its orbit and enters another orbit, it repels an electron out of orbit to repeat the action from atom to atom through the wire. The impulse that is transferred from one electron to the next determines the speed of the current.

The impulse of energy is transferred from one electron to the next almost instantaneously. The electric current travels 186,000 miles per sec.

This is the speed of light !

Billiard Balls Analogy



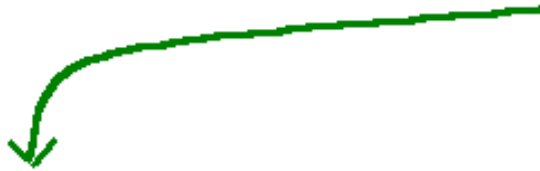
Garden Hose Analogy

(A hose filled with water)

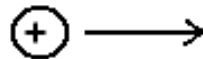
The electrons are moving together much like the water in a garden hose. When a faucet is turned *on*, one does not have to wait a noticeable time for water from the spigot to travel through the hose to the other end. The hose is already filled with water and water everywhere within the water circuit is set in motion at the same time.

Electric Current

Electric current is defined in terms of two quantities: **Direction** and **magnitude**.

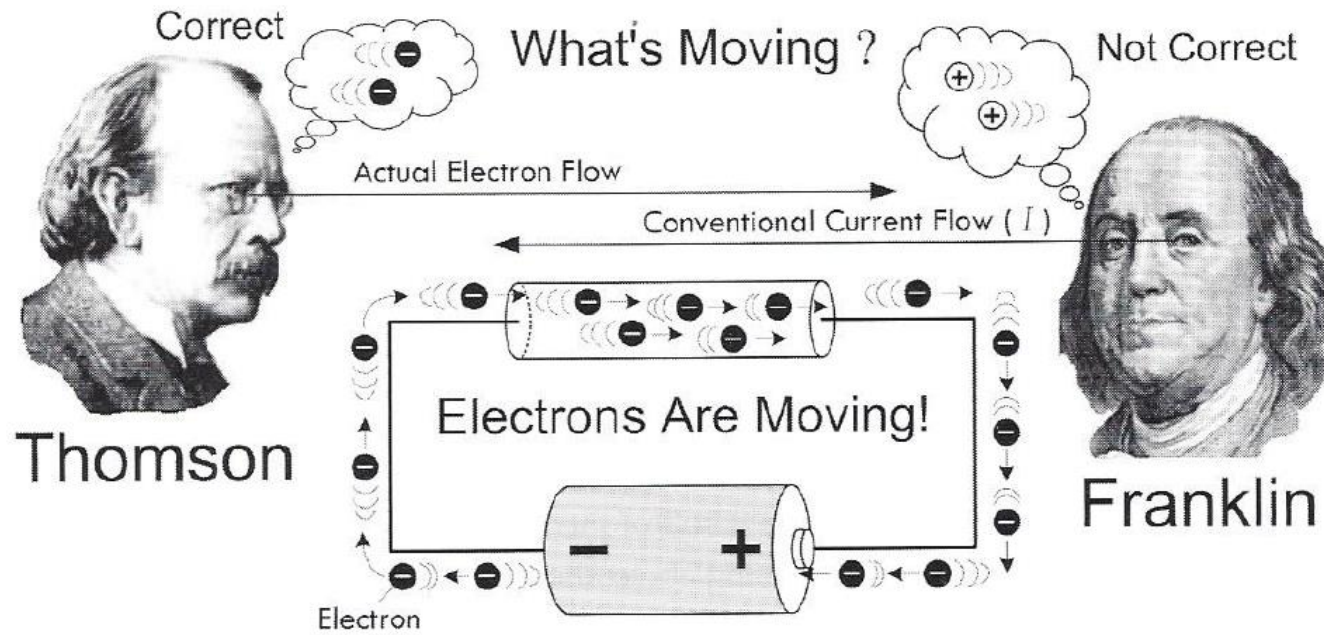


The direction of flow of electric current is conventionally represented as the direction of flow of POSITIVE charge.



This convention was initiated by Benjamin Franklin and is commonly used today in most textbooks and in practice.

But we know that electrons (with negative charge) are responsible for electric currents in metal conductors (e.g., in copper wires). Therefore, and by convention, we will assign the direction of current as opposite to the direction of flow of electrons.



Benjamin Franklin (1706-1790; was one of the Founding Fathers of the United States, and often considered the father of electronics) he had a convention of assigning positive charge to the mysterious (at that time) things that were moving and doing work in an electric circuit. This view was corrected later by the English physicist Joseph Thomson (1856 - 1940) who determined experimentally that the moving charges were negatively charged (he called them electrons), and that they were moving in the opposite direction of Franklin's conventional current. However, we got stuck with Franklin's convention.

Electric Current

Electric current is defined in terms of two quantities: Direction and magnitue.



The magnitude of current is given by the rate of change of charge past a given point in an electric circuit:

$$i(t) = \frac{dq}{dt}$$



Andre-Marie Ampere (1775-1836), a French mathematician and physicist, defined the electric current (in the 1820s) and developed a way to measure it. He was honored by having the unit of electric current, the ampere, named after him in 1881.

Electric Current

$$i(t) = \frac{dq}{dt}$$

The symbol for current is i .

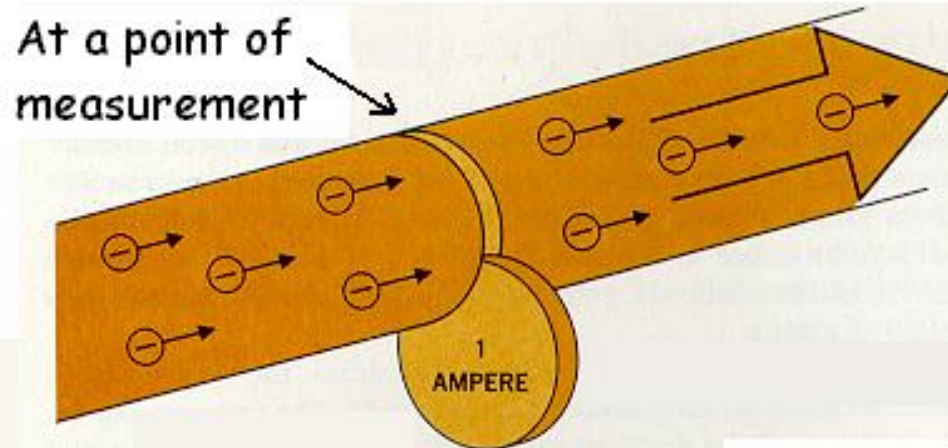
The unit of current is ampere (A); an ampere is 1 coulomb per second or 1 C/s.

The symbol for charge is q .
Its unit is the coulomb (C).

The charge on one electron is -1.602×10^{-19} C.

Thus -1 C is the charge of 6.24×10^{18} electrons

Electric Current in a wire



1 coulomb per second = **1 ampere** = 6.24×10^{18} electrons per second

How can there be a current on the order of 1 or 2 Ampere in a circuit if the drift speed is only about 1 meter per hour?

The answer is: there are many, many charge carriers **moving at once throughout the whole length** of the circuit. Current magnitude is the rate at which charge crosses a cross section of a wire on a circuit, it is not charge velocity. If the charge carriers are densely packed into the wire, then there does not have to be a high speed to have a high current.

To illustrate how densely packed the charge carriers are, we will consider a typical wire found in household lighting circuits - a 14-gauge copper wire. In a 0.01 cm thick cross-sectional slice of this wire, there would be as many as 3.51×10^{20} copper atoms. If we assume that each copper atom contributes just a single electron (the outer most electron), then there would be as much as 56.25 coulombs of charge within a thin 0.01-cm length of 14-gauge copper wire. With that much mobile charge within such a small space, a small drift speed could lead to a very large current.

Electric Current

Summary

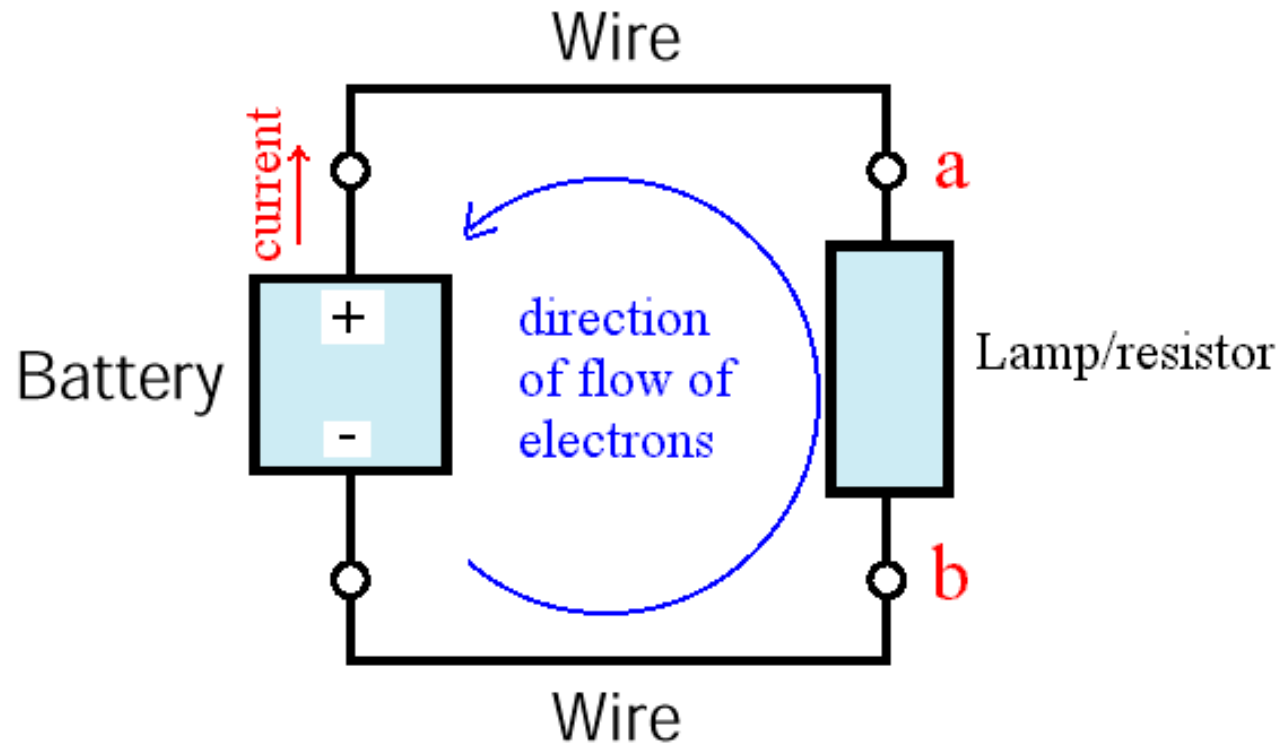
The free electrons are considered current carriers in an electric circuit

Speed: electric current, the impulse of energy that is transferred from electron to electron, travels at the speed of light.

Direction: by convention, the direction of electric current is opposite to the direction of flow of electrons.

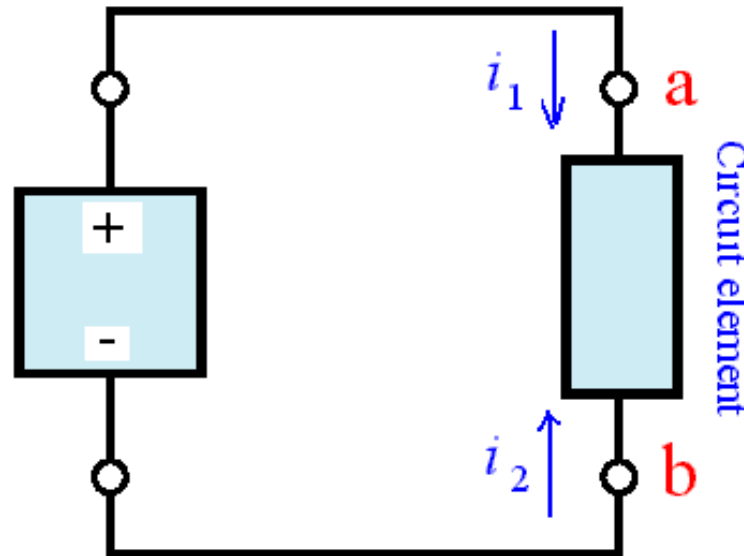
Magnitude: $i(t) = \frac{dq}{dt}$ A (or C/s)

Electric Current



Electric Current in a circuit element

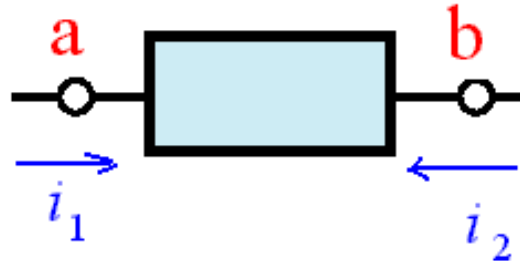
How do we label the direction of electric current in a circuit element?



Does not matter which direction is assigned on the diagram.

Note that $i_1 = -i_2$

Electric Current in a circuit element



It is important to note that the current exiting a circuit element is equal to the current entering that element.

Terminology:

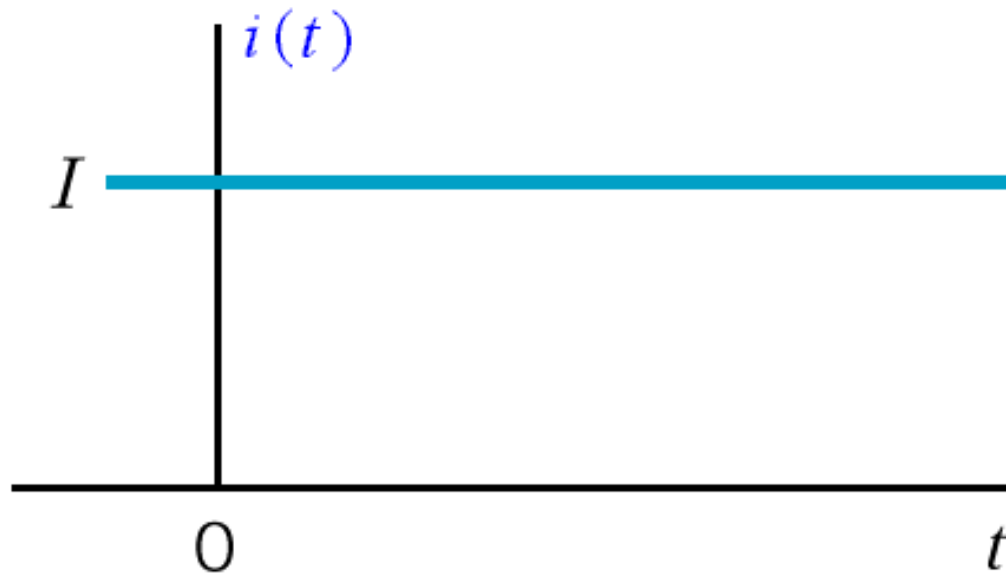
The current i_1 is the rate of flow of electric charge from terminal a to terminal b.

The current i_2 is the rate of flow of electric charge from terminal b to terminal a.

Note that $i_1 = -i_2$

Electric Current

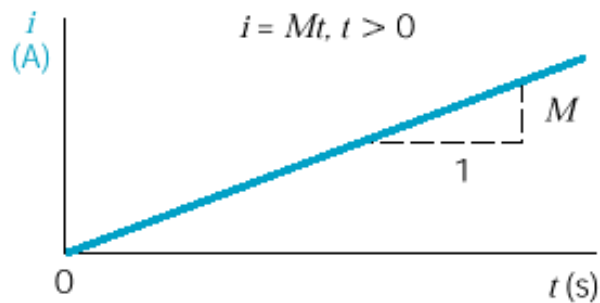
The many forms of electric current



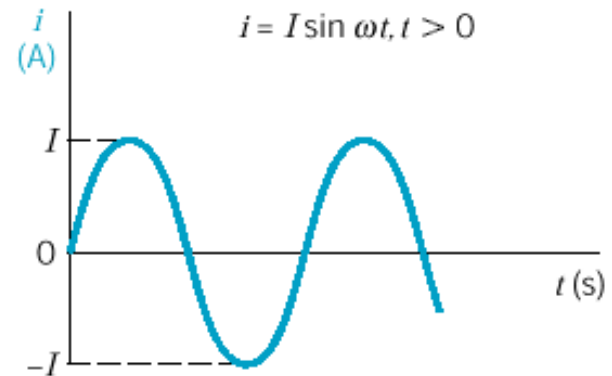
A constant current $i(t) = I$
is called a direct current (dc).

Electric Current

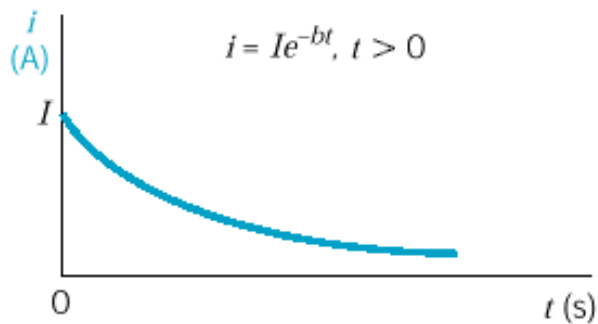
The many forms of electric current



Ramp



Sinusoid

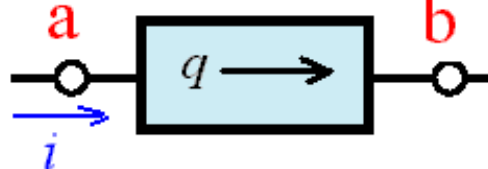


Exponential

Electric Current: Examples

Find the current past terminal a of an element when the charge that has entered the element is $q(t) = 3t - 2$ C.

Solution: $i(t) = \frac{dq}{dt} = 3$ A Current is dc.

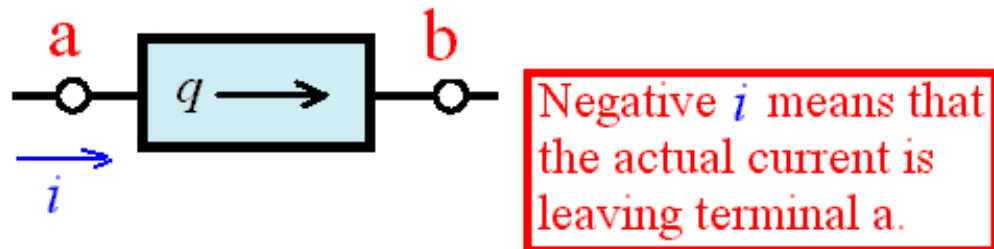


Note: The flowing charge can be +ve or -ve. Always start by assigning the current direction as that of the charge flow direction. Here, for positive charge we get the assigned current direction as the actual direction. On the other hand, if the charge is negative we get a negative current which indicates that the actual current flows in the opposite direction to the assigned one.

Find the current in a wire if the charge flowing in it is $q(t) = -3t^2$ C.

Solution:

$$i(t) = \frac{dq}{dt} = -6t \text{ A} \quad \text{Current is a ramp.}$$



Electric Charge as a Function of $i(t)$

Consider a current $i(t)$ that enters terminal a of an electric element. Derive a formula for the charge that has entered terminal a by time t .



In a statement like the above, the initial time is not specified. So, we have to start at $t = -\infty$. Starting from the formula

$$i(t) = \frac{dq}{dt} \rightarrow dq = i(t)dt$$

and integrating from $-\infty$ to t we obtain

$$\int_{q(-\infty)}^{q(t)} dq = q(t) - q(-\infty) = \int_{-\infty}^t i(\tau)d\tau$$

$$q(t) = \int_{-\infty}^t i(\tau)d\tau + q(-\infty)$$

At $t = -\infty$ the universe was not born, so the initial charge $q(-\infty) = 0$, and we arrive at the formula

$$q(t) = \int_{-\infty}^t i(\tau)d\tau$$

Electric Charge as a Function of $i(t)$ (cont.)

If we are given the total charge that had entered the element from $-\infty$ to t_0 [expressed as $q(t_0) = \int_{-\infty}^{t_0} i(\tau) d\tau$] then the above formula can be restated as

$$q(t) = \int_{t_0}^t i(\tau) d\tau + q(t_0)$$

If the problem statement is to find the (net) charge that had entered the element since time t_0 (i.e., from t_0 to t) then we set $q(t_0) = 0$,

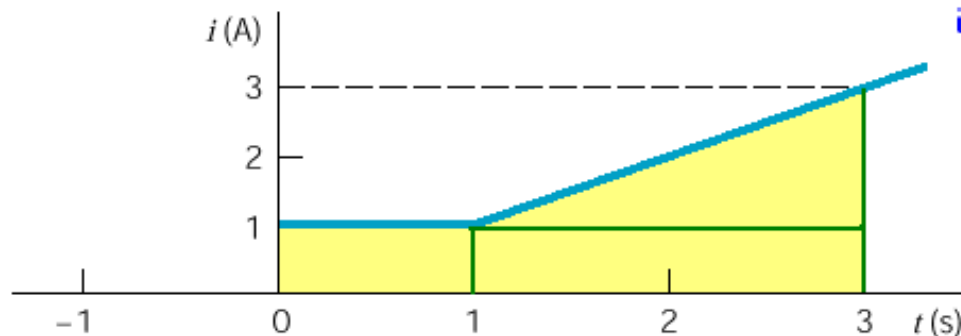
$$q(t) = \int_{t_0}^t i(\tau) d\tau$$

Electric Current

Examples



Find the charge that has entered terminal a of an element from $t=0$ to $t=3\text{s}$ when the current is



(i.e., discard
initial charge)

$\rightarrow q(0) = 0$

Solution:
$$q(3) = \int_0^3 i(t) dt + q(0) = \int_0^1 (1) dt + \int_1^3 t dt + 0 = t \Big|_0^1 + \frac{t^2}{2} \Big|_1^3 = 5 \text{ C}$$

Alternatively:

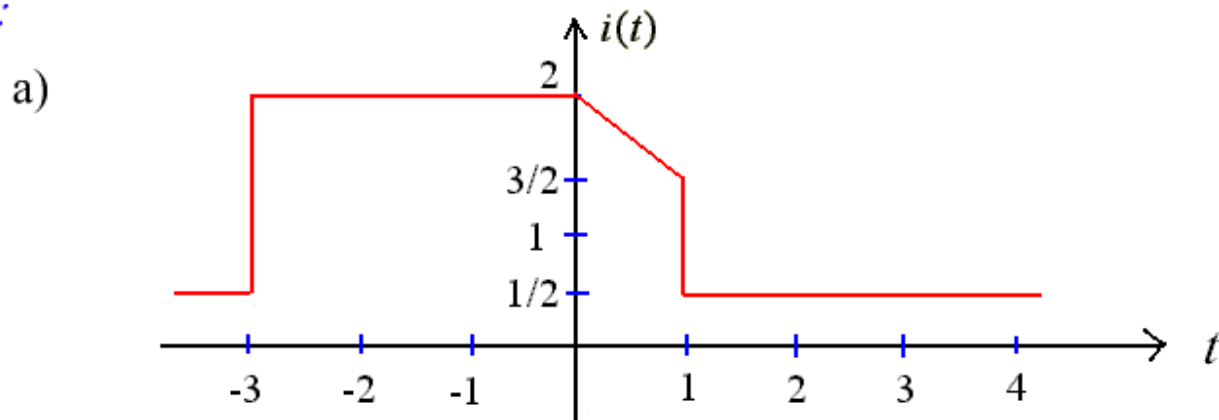
$$\begin{aligned} q(3) &= \int_0^3 i(t) dt + q(0) = \text{Area under curve}(0 < t < 3) + 0 \\ &= (1)(1) + (1)(2) + (2)(2)/2 = 5 \text{ C} \end{aligned}$$

Example: The current flowing in an element is:

$$i(t) = \begin{cases} 2 & -3 \leq t \leq 0 \\ 2 - \frac{t}{2} & 0 \leq t \leq 1 \\ \frac{1}{2} & \text{otherwise} \end{cases}$$

(a) Sketch $i(t)$ vs t . (b) determine the total charge entering the element from $t = 0$ to $t = 4$.

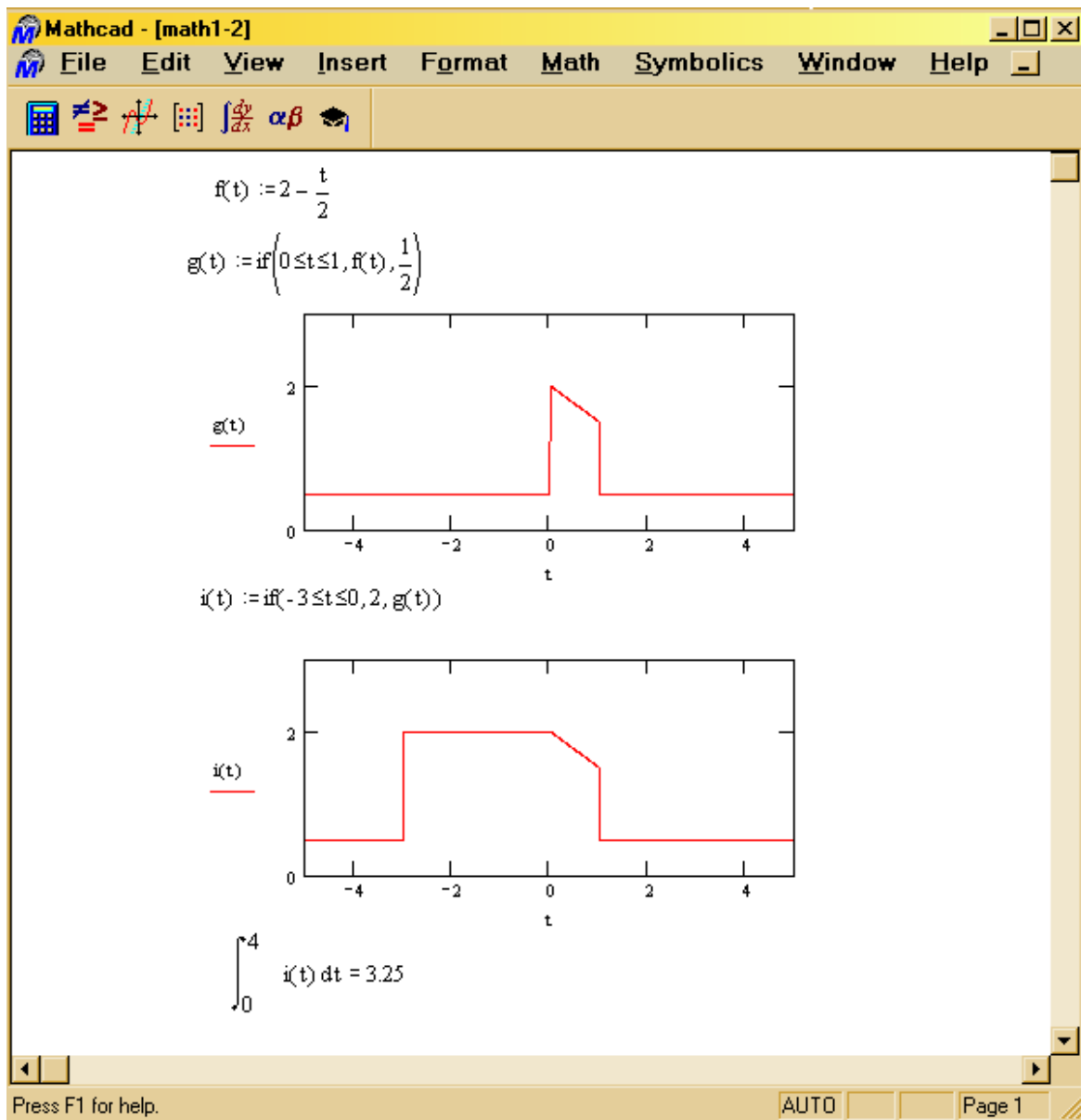
Solution:



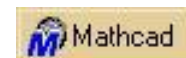
b)

$$q = \int_0^4 i(t) dt = \int_0^1 \left(2 - \frac{1}{2}t\right) dt + \int_1^4 \frac{1}{2} dt$$

$$= 2t \Big|_0^1 - \frac{1}{4}t^2 \Big|_0^1 + \frac{1}{2}t \Big|_1^4 = \underline{3.25 \text{ C}}$$



Example (Cont.)
Mathcad solution



Example

A current of $8 \mu\text{A}$ flows through a wire.
How many coulombs of charge have passed
through the wire in 1ms ?

Solution: $1 \text{ ms} = 0.001 \text{ s}$
 $I = 8 \mu\text{A} = 8 \times 10^{-6} \text{ A}$

The total charge Q is given by:

$$Q = \int_0^{0.001} I d\tau = I t \Big|_0^{0.001} = 8 \times 10^{-6} \times 0.001 = 8 \times 10^{-9} \text{ C}$$

or $Q = 8 \text{ nC}$

for constant
current

$$Q = I \times \Delta t$$

Example



Find the charge that had entered terminal *a* of a circuit element by time *t* when the current is specified by

$$i(t) = \begin{cases} 0 & t < 0 \\ Mt & t \geq 0 \end{cases}$$

Solution:

$$q(t) = \int_{-\infty}^t i(\tau) d\tau = \int_{-\infty}^0 (0) d\tau + \int_0^t M\tau d\tau = \frac{1}{2} M\tau^2 \Big|_0^t = \frac{1}{2} Mt^2$$

Your turn: Repeat the above example for $i(t) = \sin(2\pi t)$ (Amp) for $t \geq 0$ and $i(t) = 0$ for $t < 0$. Evaluate $q(0.5)$ and $q(1)$.

Systems of Units

SI Base Units

Quantity	SI Unit	
	Name	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

Systems of Units

Derived Units in SI

Quantity	Unit Name	Formula	Symbol
Acceleration—linear	meter per second per second	m/s^2	
Velocity—linear	meter per second	m/s	
Frequency	hertz	s^{-1}	Hz
Force	newton	$\text{kg} \cdot \text{m/s}^2$	N
Pressure or stress	pascal	N/m^2	Pa
Density	kilogram per cubic meter	kg/m^3	
Energy or work	joule	$\text{N} \cdot \text{m}$	J
Power	watt	J/s	W
Electric charge	coulomb	$\text{A} \cdot \text{s}$	C
Electric potential	volt	W/A	V
Electric resistance	ohm	V/A	Ω
Electric conductance	siemens	A/V	S
Electric capacitance	farad	C/V	F
Magnetic flux	weber	$\text{V} \cdot \text{s}$	Wb
Inductance	henry	Wb/A	H

Systems of Units

SI Prefixes

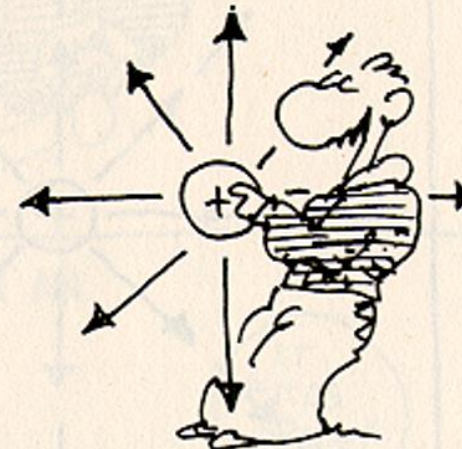
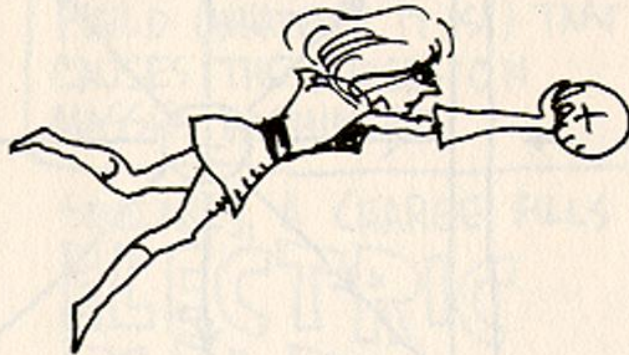
Multiple	Prefix	Symbol
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

Voltage

The basic variables in an electric circuit are current and voltage.

Voltage describes the energy required to cause a charge to move across the terminals of a circuit element.

HERE RINGO HOLDS A POSITIVE CHARGE, AND, STARTING FAR AWAY, I BRING A SMALL POSITIVE TEST CHARGE IN CLOSE TO IT.



AS I MOVE IN, THE CHARGE IS REPELLED, SO I HAVE TO EXERT FORCE TO PUSH IT CLOSER. FORCE TIMES DISTANCE EQUALS **WORK** = I DO WORK ON THE TEST CHARGE.

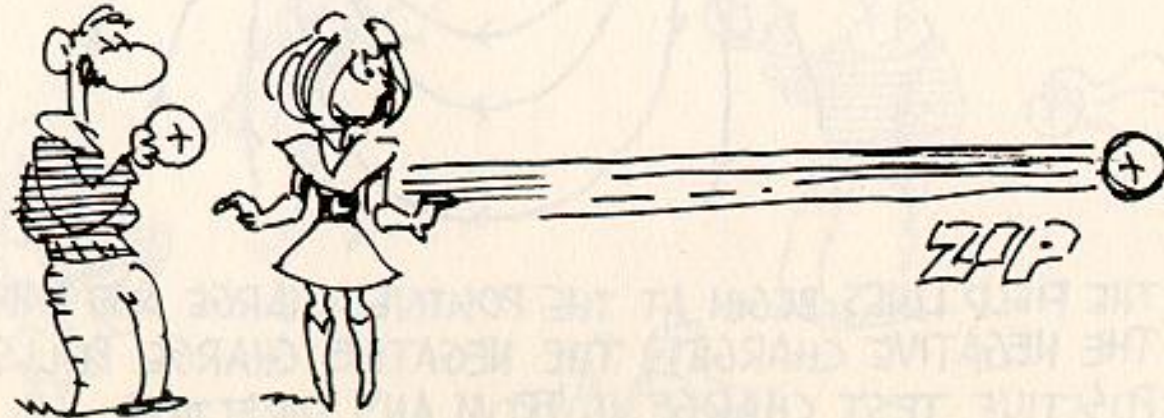
The positive charge is moving against the electric field created by the charge pair and it requires work (energy) to be exerted to do that. The energy is stored in the positive charge as electric potential. This is analogous to lifting a mass from ground where the work exerted (energy) is stored as gravitational potential energy in the mass.

WE SAY THAT THE WORK
GOES INTO THE

POTENTIAL ENERGY

OF THE TEST CHARGE.

IF I RELEASE THE CHARGE, IT
FLIES AWAY, AND POTENTIAL ENERGY
IS CONVERTED INTO KINETIC ENERGY.



WE WOULD LIKE TO ATTRIBUTE THE POTENTIAL ENERGY SOLELY TO THE ELECTRIC FIELD OF RINGO'S CHARGE, SO WE DIVIDE OUT MY TEST CHARGE AND WRITE:

$$\text{Potential} = \frac{\text{POTENTIAL ENERGY}}{\text{CHARGE}}$$



THIS EQUATION DEFINES A NEW QUANTITY, THE ELECTRIC POTENTIAL.* POTENTIAL MEASURES ENERGY PER CHARGE. ITS UNITS ARE JOULES PER COULOMB, WHICH WE GIVE A NAME ALL ITS OWN, THE **VOLT**.

$$1 \text{ Volt} = 1 \frac{\text{JOULE}}{\text{COULOMB}}$$



IF A BATTERY IS RATED
AT 6 VOLTS, THAT
MEANS IT IS PREPARED
TO GIVE 6 JOULES
OF ENERGY TO EVERY
COULOMB THAT IS
MOVED FROM ONE
TERMINAL TO THE
OTHER.

Analogy to Gravitational Potential/Energy

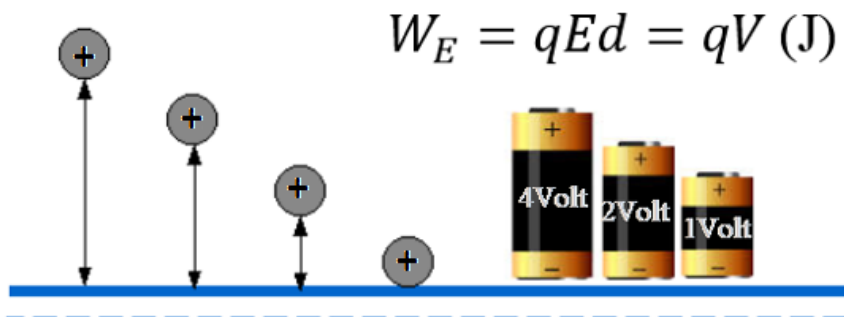
Electric Potential ($V=Ed$)

Gravitation Potential (gh)



Electric Potential Energy

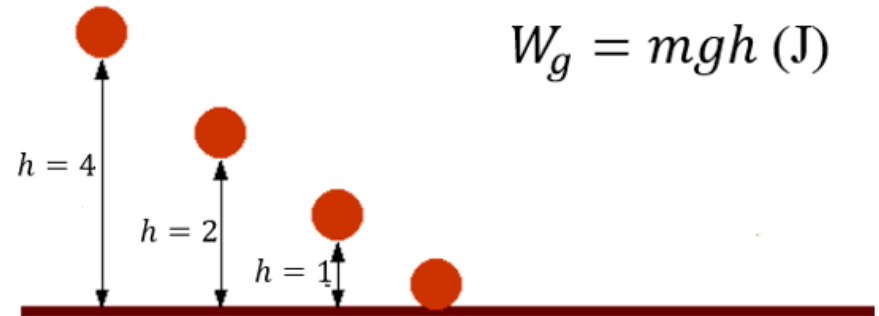
+++++



$$W_E = qEd = qV \text{ (J)}$$

Electric Field (E)

Gravitational Potential Energy



$$W_g = mgh \text{ (J)}$$

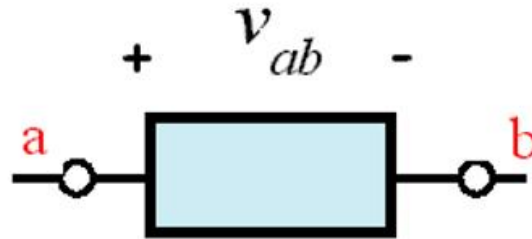
Gravity Field

Let's make a parallel comparison of Gravitational Potential Energy to Electric Potential Energy

$$W_g = mgh \quad \text{Units: } kg \cdot \frac{N}{kg} \cdot m = Nm = J$$

$$W_E = qEd \quad \text{Units: } C \cdot \frac{N}{C} \cdot m = Nm = J$$

Voltage



The voltage across an element is the work (energy w) required to move a unit positive charge from the - terminal to the + terminal. The unit of voltage is the Volt, V.

$$v = \frac{dw}{dq}$$

When work is linear
the formula reduces to: $\rightarrow$
(here, Q is total charge
and w is total work).

$$v = \frac{w}{Q}$$

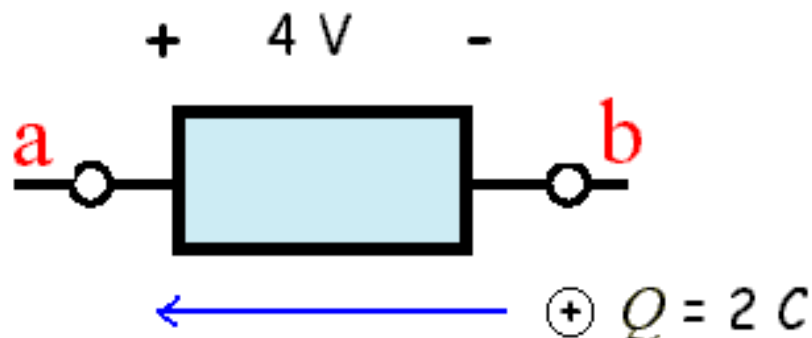
volt = joule/coulomb or $V = J/C$

The above definition is also valid for "... to move a unit negative charge from the + terminal to the - terminal."

Voltage

Examples

$$v = \frac{w}{Q}$$



Find the energy required to move 2 coulombs of charge through a constant voltage of 4 volts.

Solution:

$$\text{energy} = (\text{voltage}) \times (\text{total charge}) = 4 \times 2 = 8 \text{ J}$$

Voltage

Examples

$$v = \frac{w}{Q}$$

The average current in a typical lightning thunderbolt is 20 kA and its typical duration is 0.1 s. The voltage between the clouds and the ground is 500 MV. Determine the total charge transmitted to earth and the energy released.

Solution:

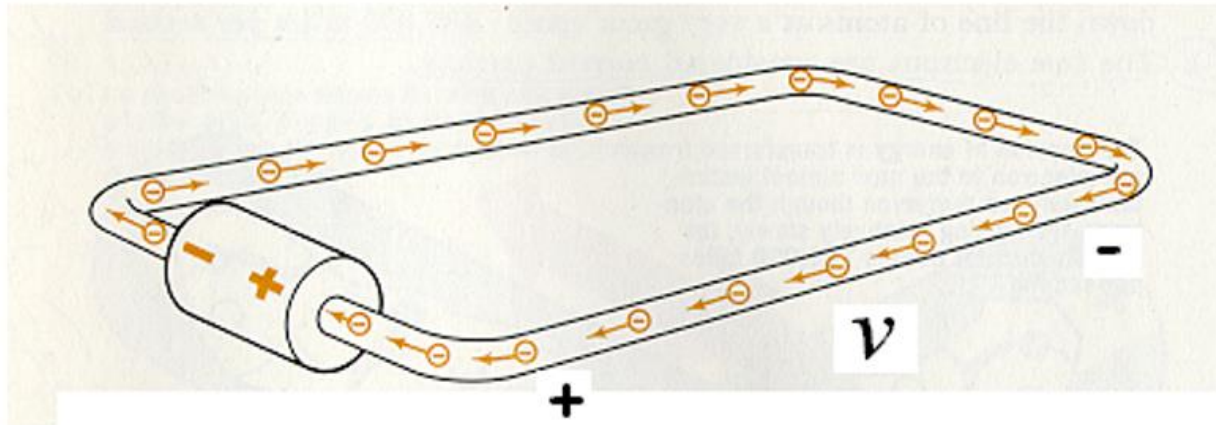
The total charge is (assuming a constant current)

$$Q = I \cdot \Delta t = 20,000 \times 0.1 = 2000 \text{ C}$$

The total energy released is (assuming a constant voltage)

$$w = Q \cdot v = 2000 \times (500 \times 10^6) = 10^{12} \text{ J} = 1 \text{ TJ}$$

Voltage Drop Across a Short Stretch of Copper Wire



What is the value of voltage v in this circuit?

Solution:

It takes (almost) no work (energy) to move the free electrons in the copper wire. Therefore,

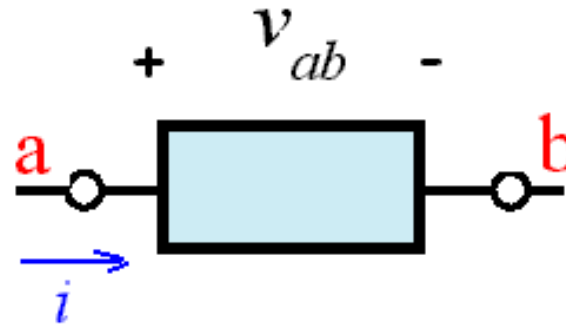
$$v = \frac{w}{Q} = 0$$

The voltage across any stretch of (ideal) wire in a circuit is ZERO.

Voltage

When a current flows through a circuit element, it develops a voltage drop across the terminals of that element.

Voltage describes the energy required to cause a charge to move across the terminals of a circuit element.

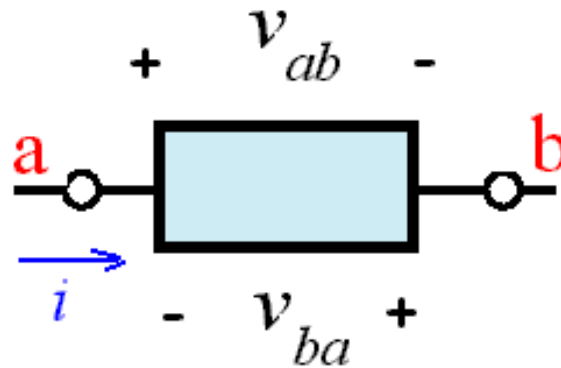


v_{ab} can be read as "the voltage at terminal a with respect to the voltage at terminal b."

Voltage

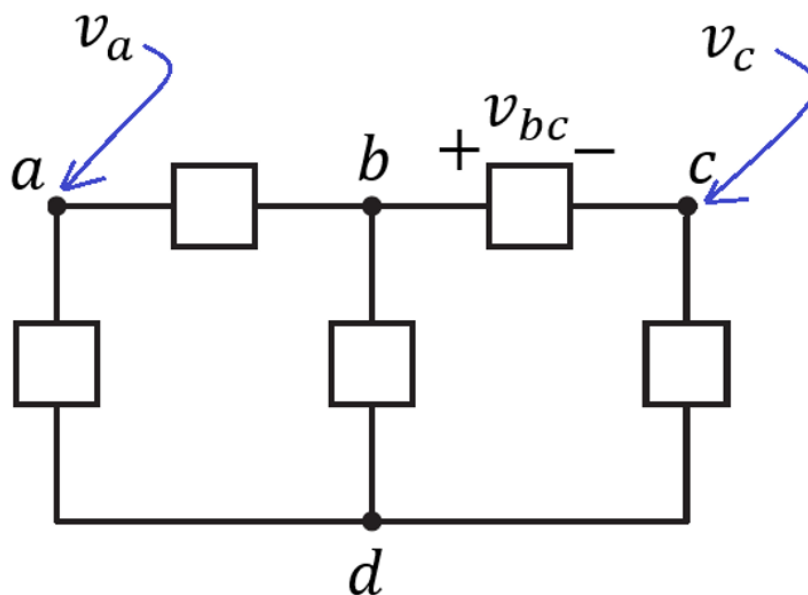
The direction of voltage is given by its polarity. Voltage can be positive or negative. The figure below shows two ways of defining the voltage across the element. Here,

$$v_{ab} = - v_{ba}$$



The Voltage Reference Point

We must always be clear about the reference point for a voltage in a circuit. In circuit analysis, we normally define a reference point (node) in the circuit. Let us label such reference node as d . If we drop the second subscript and write v_a at node a in the circuit (see figure below), then $v_a = v_{ad}$.

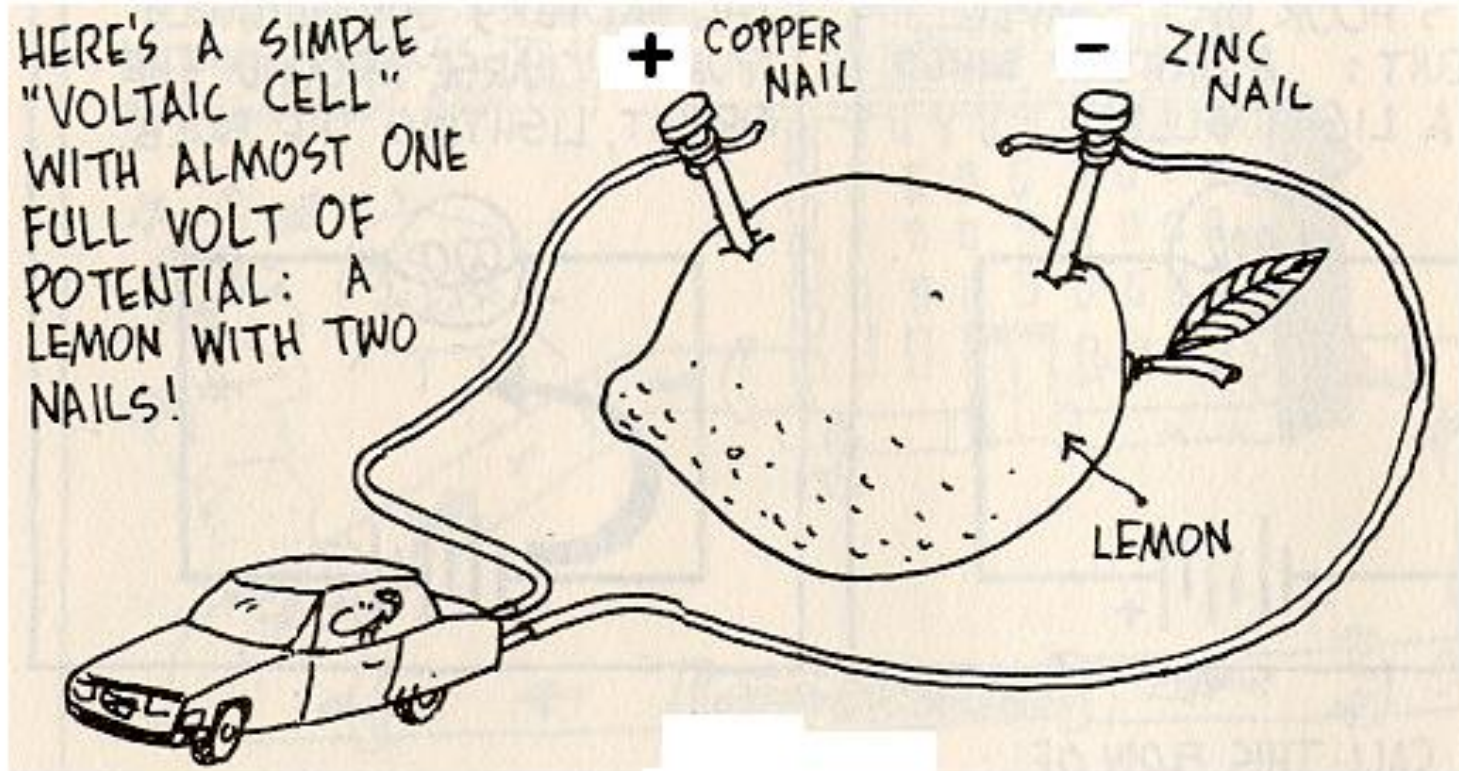


Electric Battery

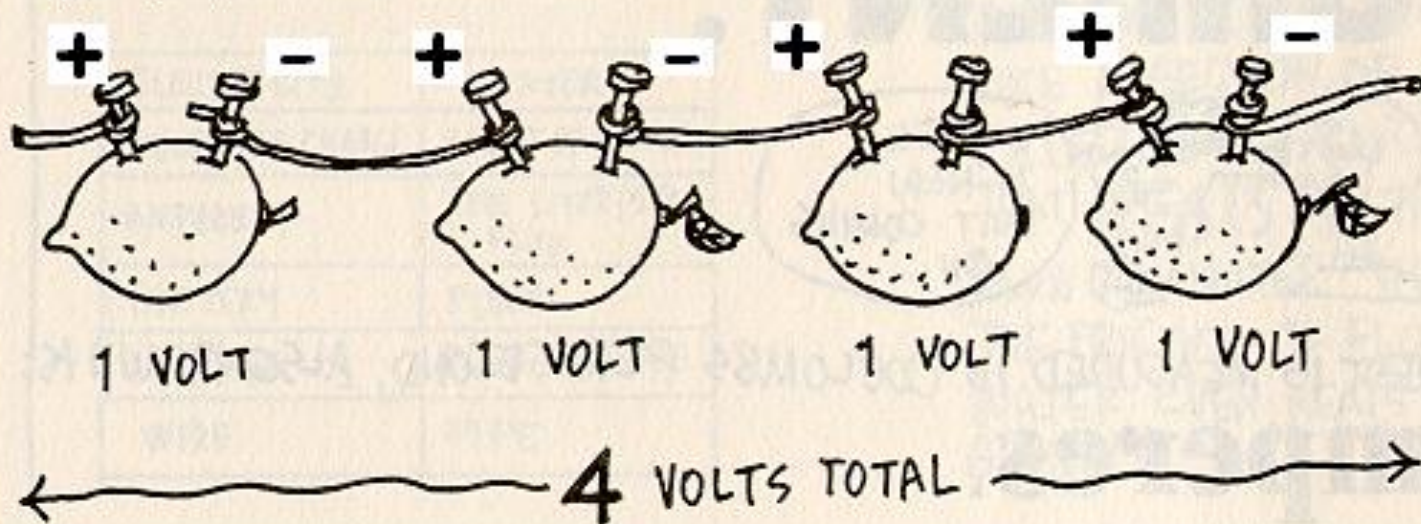
The unit of voltage, the volt, was named in honor of Alessandro Volta. Volta invented the electric battery in 1794. The battery provided the first continuous flow of electricity.



Alessandro Antonio Volta
(1745-1827) was an Italian
physicist.

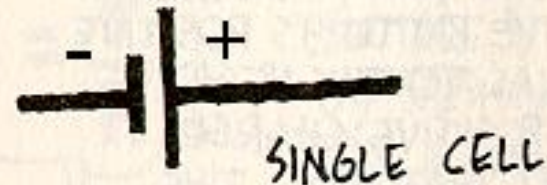


VOLTA ALSO FOUND THAT BY CONNECTING CELLS IN SERIES,
THE POTENTIALS ADD UP TO GIVE LARGE VOLTAGES:

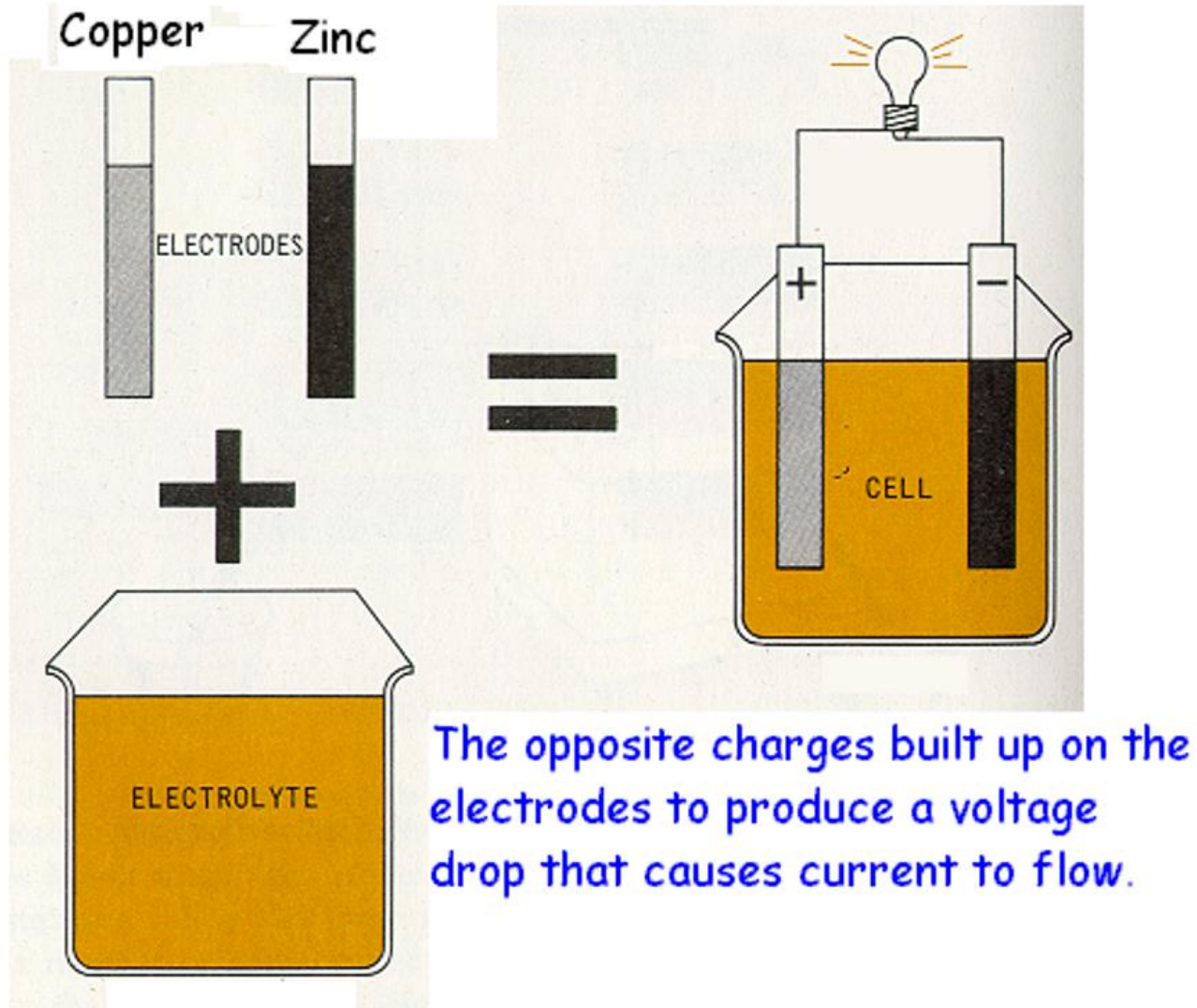


Battery Symbol (constant voltage)

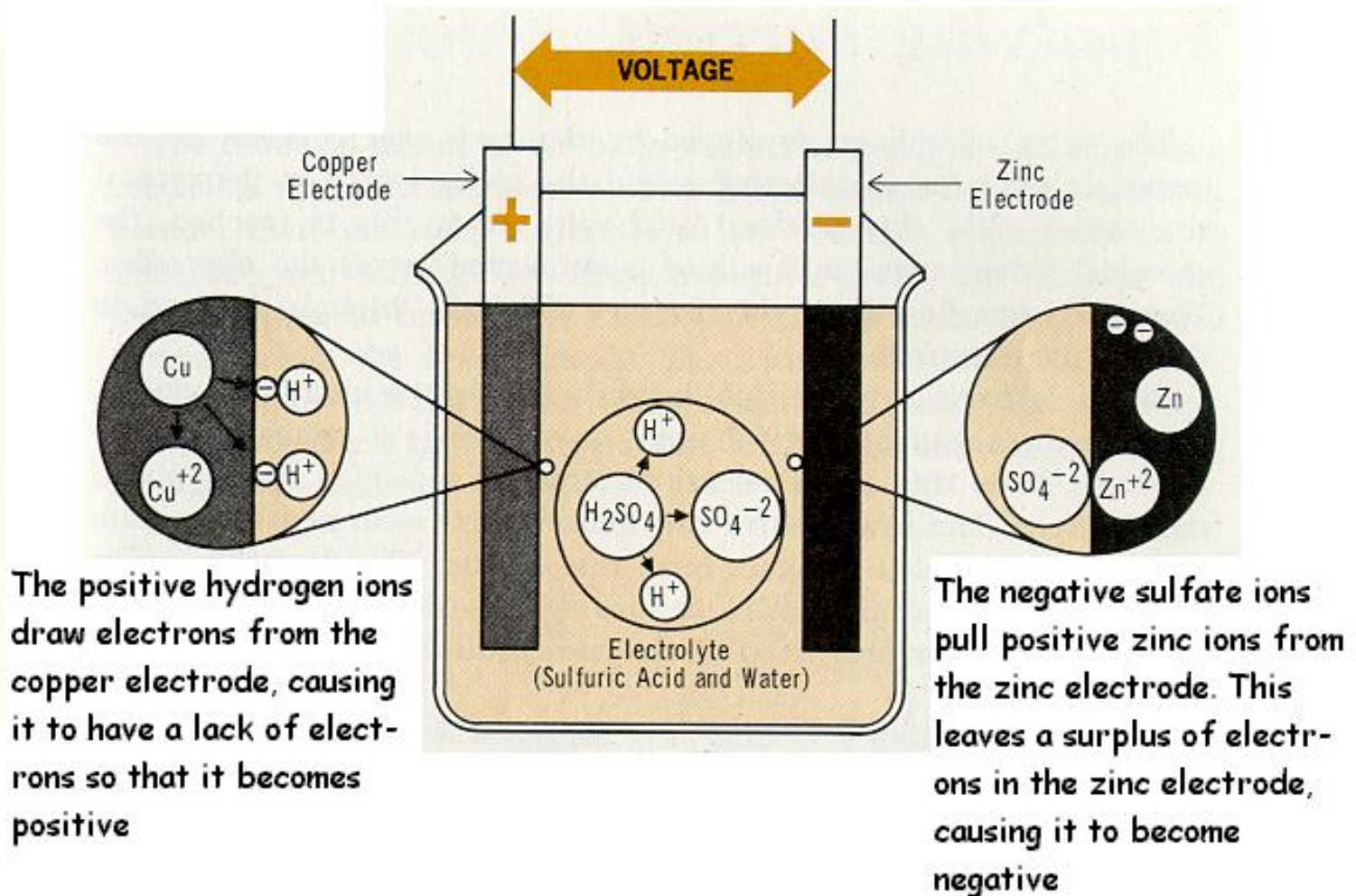
A FLASHLIGHT "BATTERY" IS ACTUALLY A SINGLE CHEMICAL CELL. A TRUE BATTERY, LIKE THE ONE IN YOUR CAR, CONSISTS OF SEVERAL CELLS CONNECTED IN SERIES, AS ABOVE. THEIR ELECTRICAL SYMBOLS ARE:



Basic Wet Cell Battery

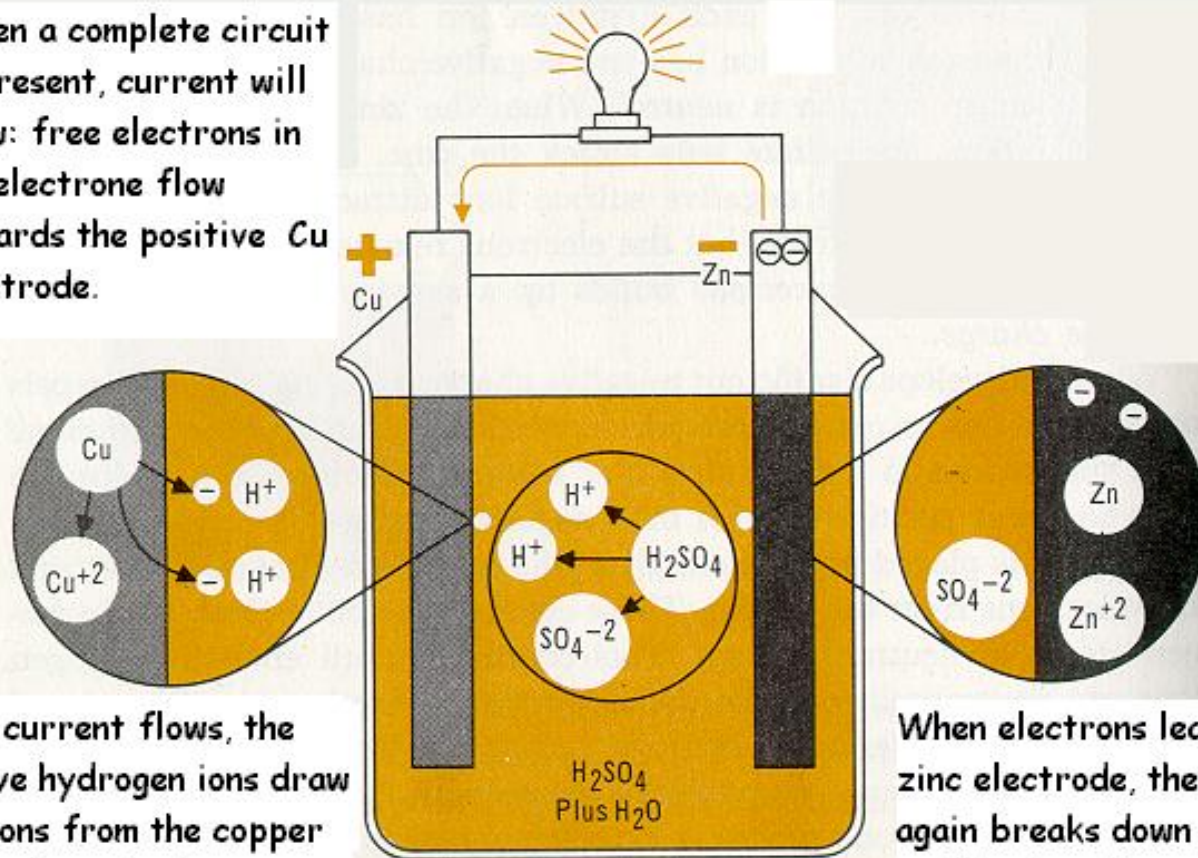


Developing a Voltage



Supplying Current

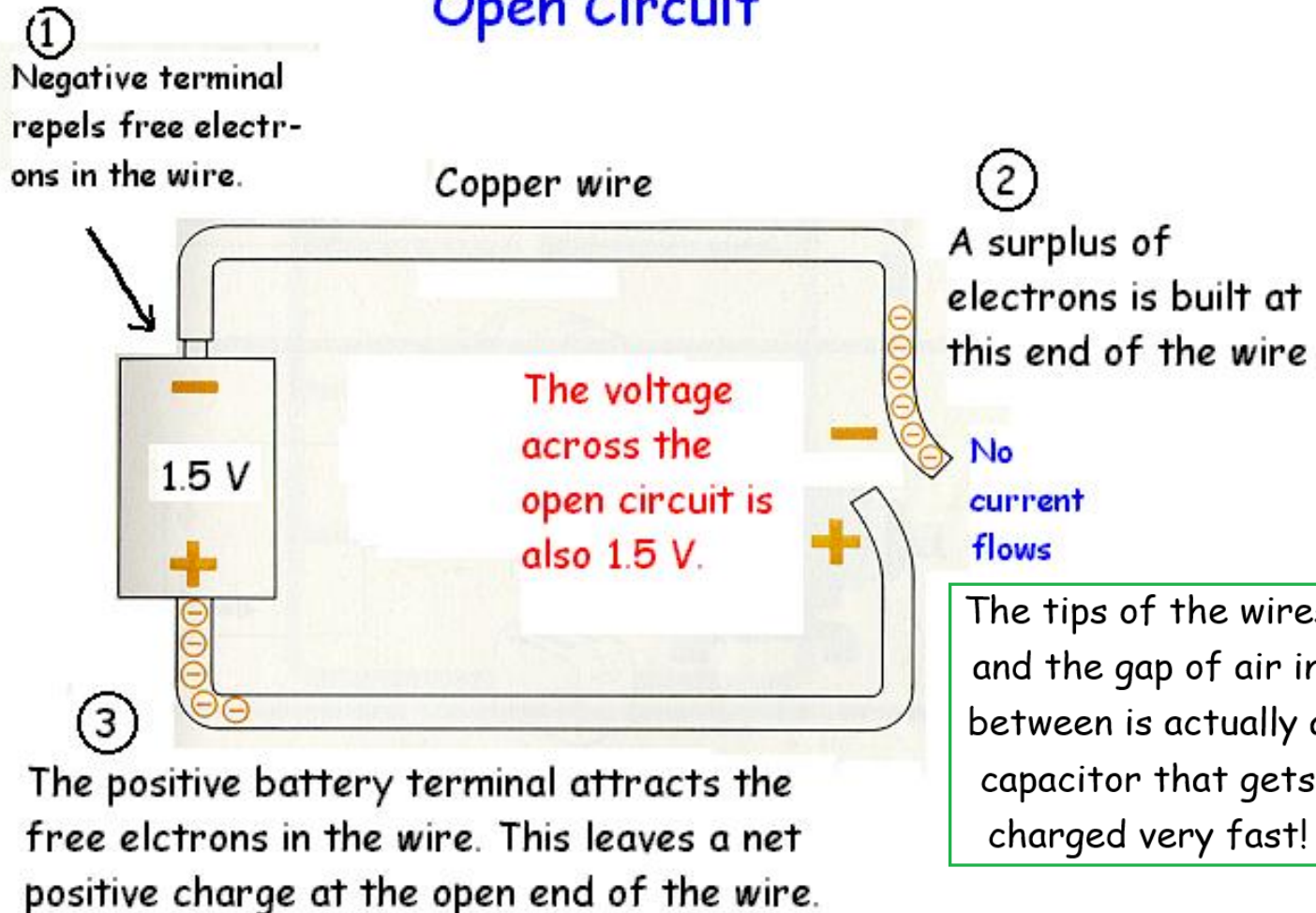
When a complete circuit is present, current will flow: free electrons in Zn electrode flow towards the positive Cu electrode.



When current flows, the positive hydrogen ions draw electrons from the copper electrode as electrons enter the copper from the circuit.

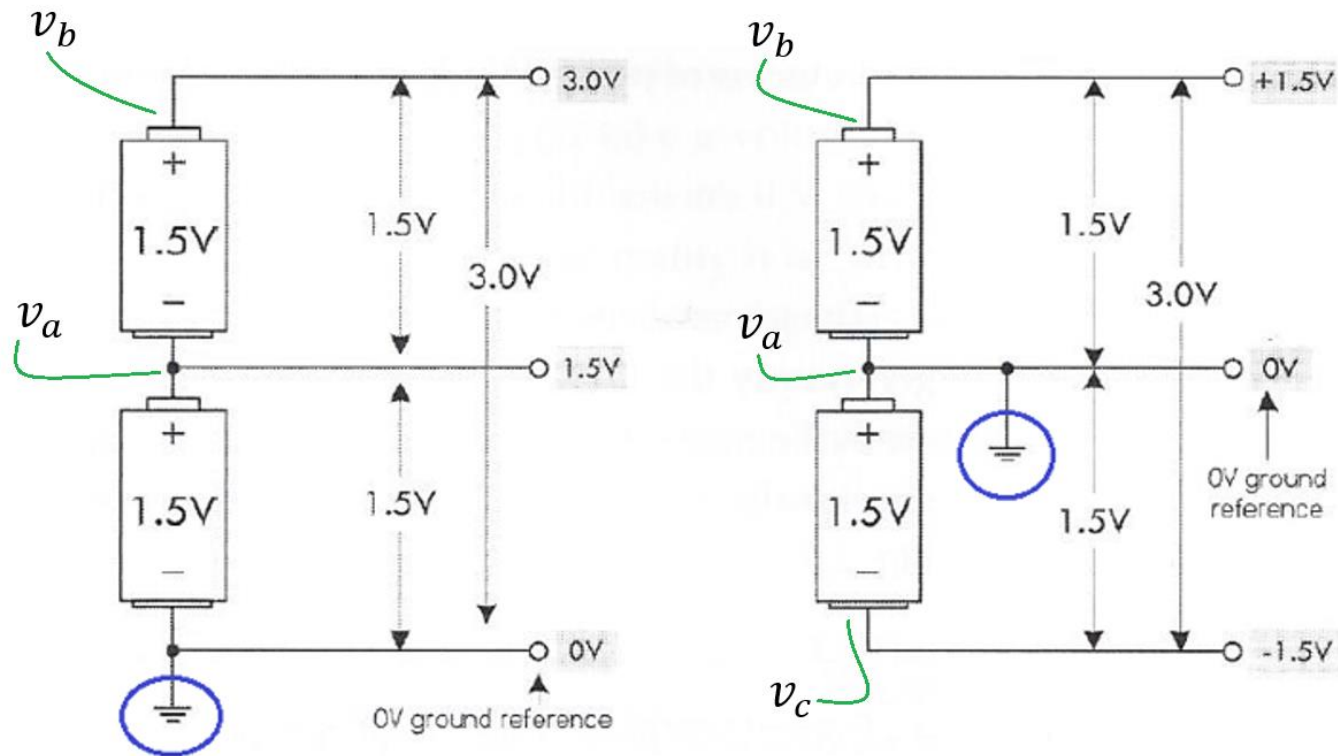
When electrons leave the zinc electrode, the zinc again breaks down into positive ions to combine with the negative sulfate ions. This produces more free electrons in the Zn.

Charge Distribution in an Open Circuit



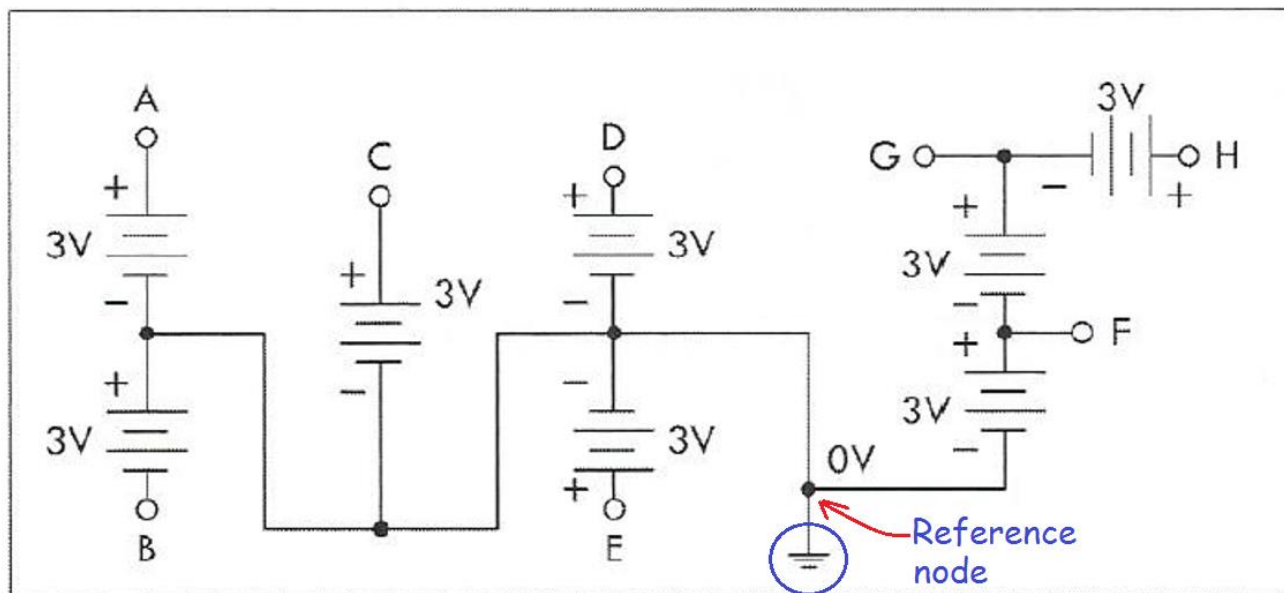
Combining Batteries

The reference point or "ground" is very critical when measuring voltage drops



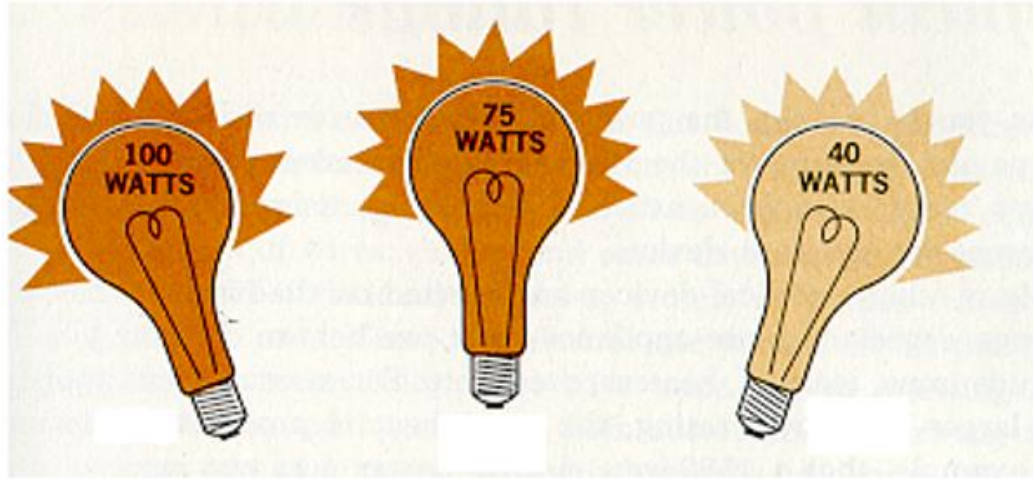
Combining Batteries

Example: Find the voltages (referenced to ground) at the various locations indicated in the following figures.



Answer: $A = 3 \text{ V}$, $B = -3 \text{ V}$, $C = 3 \text{ V}$, $D = 3 \text{ V}$, $E = 3 \text{ V}$,
 $F = 3 \text{ V}$, $G = 6 \text{ V}$, $H = 9 \text{ V}$.

Power and Energy



The wattage , or power , rating of a lamp tells how bright the lamp will be when used in a circuit. Actually, it measures the heating of the lamp filament.

Next, we will learn how to compute the power and energy delivered (or absorbed) by a circuit element.

Power and Energy

Power is the time rate of expending or absorbing energy.

$$p(t) = \frac{dw}{dt}$$

where p is power in watts, w is energy in joules, and t is time in seconds.

Expression for Energy as a function of power

If we integrate the expression $p(t) = \frac{dw}{dt}$ we would obtain the formula for the energy as a function of power

$$w(t) = \int_{t_0}^t p(\tau) d\tau + w(t_0)$$

t_0 is the reference/initial time of interest and $w(t_0)$ is the initial energy. Normally, we assume the reference time is zero and the initial energy is zero which reduces the above expression to

$$w(t) = \int_0^t p(\tau) d\tau$$

Instantaneous Power and Average Power

The power as defined by $p(t) = \frac{dw}{dt}$ represents the *instantaneous power* or the power at the instant of time t . $p(t)$ is a function of time.

We may also define the *average power* P_{ave} or simply P as the average of the instantaneous power over a period of interest, say $t_0 \leq t < t_1$, as

$$P = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} p(t) dt$$

P is a constant. Let Δw be the total energy expended over a period of time $\Delta t = t_1 - t_0$, then we may evaluate the average power by simply using the formula

$$P = \frac{\Delta w}{\Delta t} = \frac{w(t_1) - w(t_0)}{t_1 - t_0}$$

Average Power of a Periodic $p(t)$

If the **instantaneous power** $p(t)$ is periodic with period T , then the **average power** is given by the formula

$$P = \frac{1}{T} \int_0^T p(t) dt$$

The above formula is the average power over a full period of $p(t)$. It is also the average power of $p(t)$ for all time! (**Your turn:** Prove it)

Hint: In the above definition of average power, T can be any integer multiple of the fundamental period of $p(t)$.

Your turn: Determine the average power if the instantaneous power is $p(t) = \cos(2\pi t)$. Repeat for $p(t) = \cos^2(2\pi t)$. Is $\cos^2(2\pi t)$ periodic? If yes, what is its period?

What is the average power if the instantaneous power is constant?

If $p(t) = a$ for all time, where a is a constant, then the average power is $P = a$. This fact is intuitively simple, but it can be proved.

Your turn: Prove it.

(Hint: treat $p(t)$ as a constant a for $-\alpha < t < +\alpha$ and employ the general integral for average power

$$P = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} p(t) dt = \frac{1}{\alpha - (-\alpha)} \int_{-\alpha}^{+\alpha} a dt$$

Integrate and then take the limit of your answer as $\alpha \rightarrow \infty$.

Instantaneous Power and Average Power: Examples

Determine the instantaneous power and the average power over the interval $(t_0 \leq t \leq t_1)$ if the energy is given by:

a. $w(t) = a$ (J)

b. $w(t) = at$ (J)

c. $w(t) = ae^{-bt}$ (J)

Solution:

a. $p(t) = \frac{da}{dt} = 0$ (W). $P = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} (0) dt = 0$ (W)

b. $p(t) = \frac{dat}{dt} = a$ (W). $P = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} a dt = \frac{1}{t_1 - t_0} at \Big|_{t_0}^{t_1} = \frac{at_1 - at_0}{t_1 - t_0} = a$ (W)

c. $p(t) = \frac{dae^{-bt}}{dt} = -abe^{-bt}$ (W). $P = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} -abe^{-bt} dt$
$$= \frac{1}{t_1 - t_0} ae^{-bt} \Big|_{t_0}^{t_1} = \frac{ae^{-bt_1} - ae^{-bt_0}}{t_1 - t_0}$$
 (W)

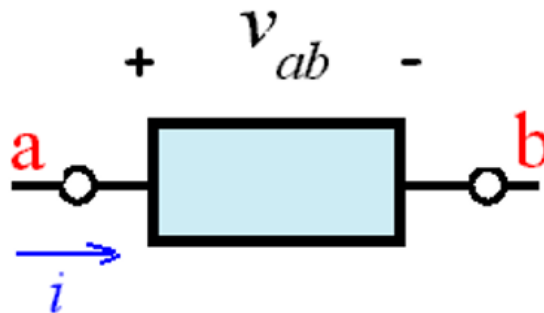
Note: We could also have used the more direct formula $P = \frac{\Delta w}{\Delta t} = \frac{w(t_1) - w(t_0)}{t_1 - t_0}$

Power in the Context of a Circuit Element

$$p(t) = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt} = v \cdot i$$

$$p = v \cdot i$$

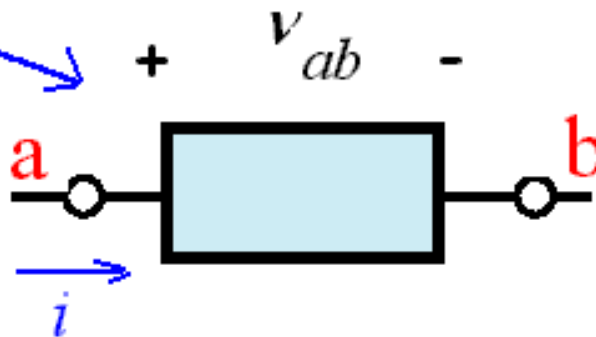
The unit of power is Watt or
Joule/Sec or (Volt)(Amp)



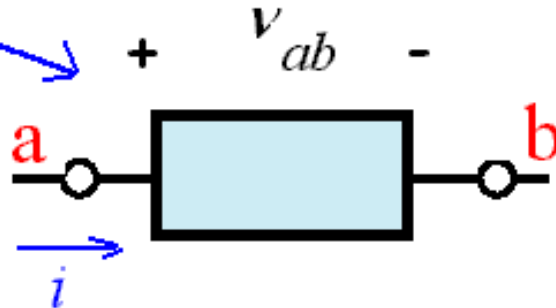
How do we know if a circuit element is delivering power or absorbing power?

The sign of p determines that. But for that to work, we need to use the so called "passive convention."

The assigned direction of the current is directed from the + terminal of the voltage to the - terminal.



The assigned direction of the current is directed from the + terminal of the voltage to the - terminal.



Terminology:

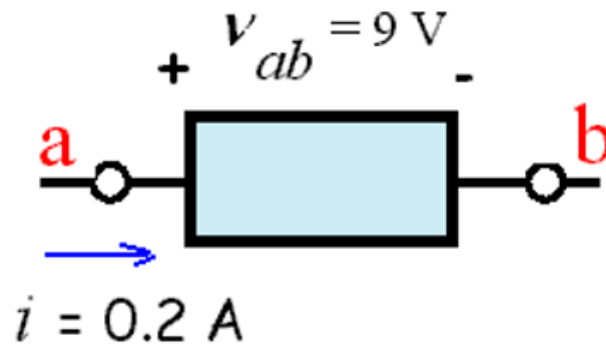
Power absorbed is also called "power dissipated by the element" or "power delivered to the element."

"Power delivered by the element" is also be called "power supplied by the element."

With this convention,
 $p = v \cdot i$ is the power *absorbed* by the element if $p > 0$.
If $p < 0$, then the power is *delivered* by the element.

Power/Energy Examples

Consider the element shown below. Find the power and tell if it is actually absorbed by the element *or* delivered by the element.



Note that when the current and voltage are constants, then the instantaneous power is constant and in this case the average power is equal to the instantaneous power.

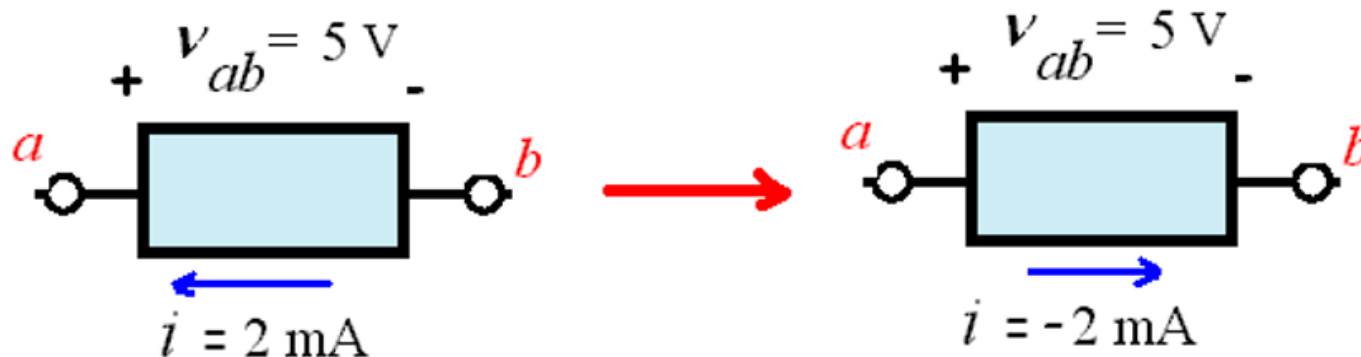
Solution:

Since current direction and voltage polarity adhere to the passive convention, then $p = 9 \times 0.2 = 1.8 \text{ W}$

P is positive, therefore power is absorbed by the element.

Power/Energy Examples

Consider the element shown below. Find the power and tell if it is actually absorbed by the element *or* delivered by the element.



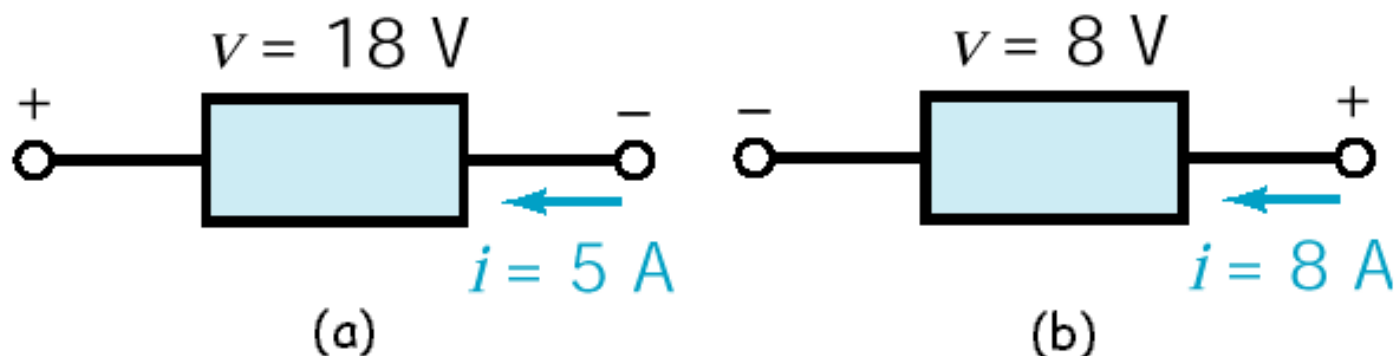
Solution:

Since current direction and voltage polarity does not adhere to the passive convention, *we first reverse the direction of the current and change its sign*. Passive convention is now met:

$$p = 5 \times (-0.002) = -0.01\text{ W}$$

$p < 0$, therefore power is delivered (supplied) by the element.

Example: Calculate the power absorbed or supplied by each element.



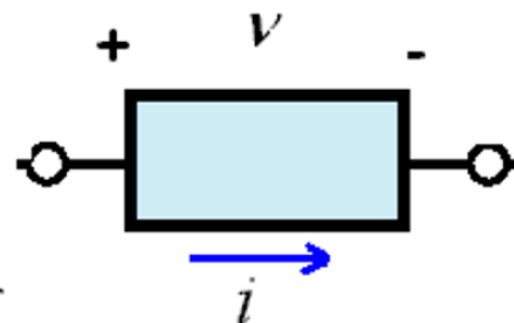
Solution:

- (a) $P = 18 \times (-5) = -90 \text{ W}$; power is supplied by the element
- (b) $P = 8 \times 8 = 64 \text{ W}$; power is absorbed by the element.

Power/Energy Examples

Consider the circuit element shown in the following figure with

$$v = -8e^{-t} \text{ V} \quad \text{and} \quad i = 20e^{-t} \text{ A}$$



Assume the current and voltage to be zero for $t < 0$. Determine the expression for the energy $w(t)$ and find $w(1)$.

Negative for all $t > 0$

Solution:

$$p = vi = (-8e^{-t})(20e^{-t}) = -160e^{-2t} \text{ W}$$

This element is providing energy to the charge flowing through it.

Energy is the integral of power

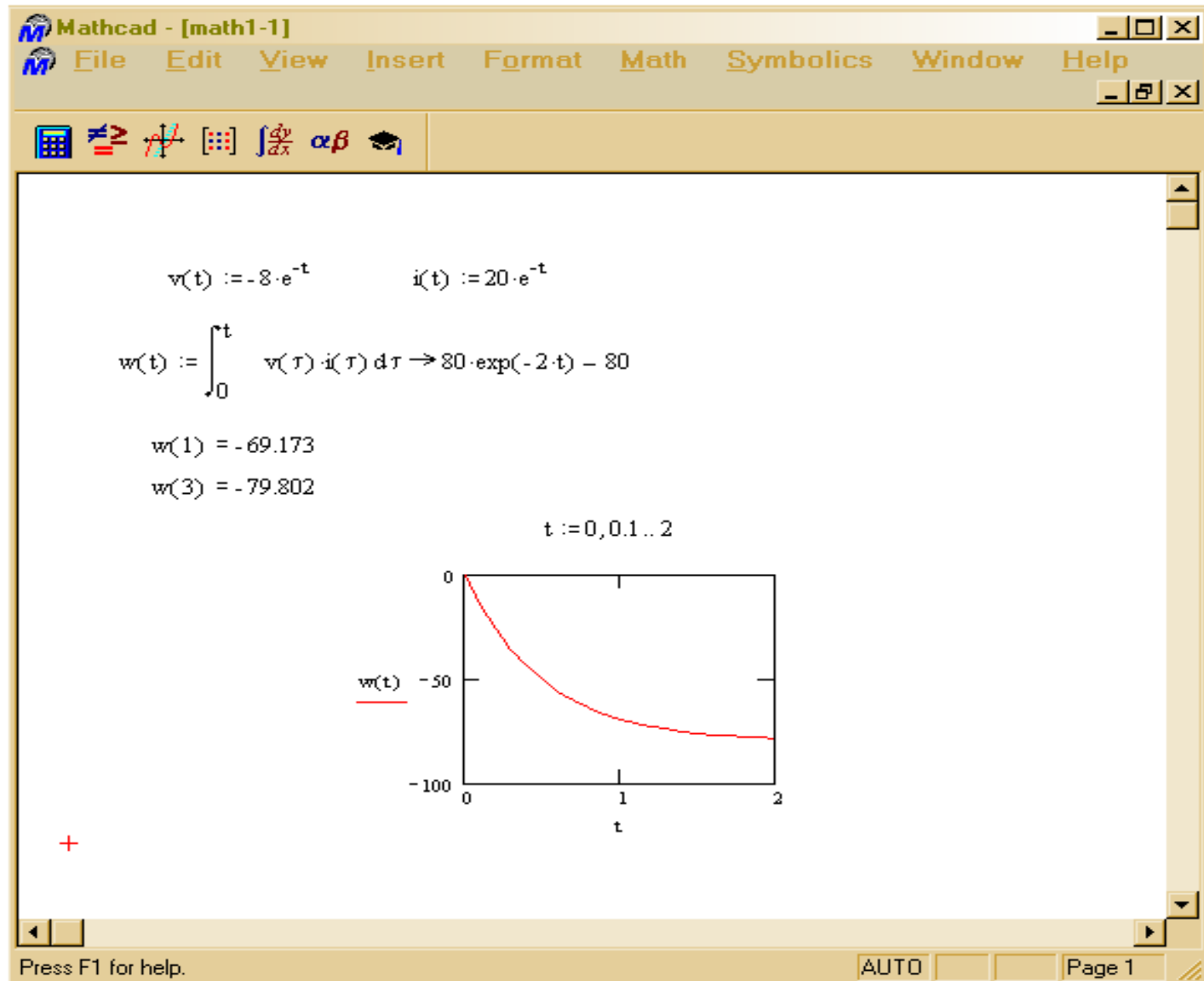
$$w(t) = \int_0^t p d\tau = \int_0^t -160e^{-2\tau} d\tau = -160 \frac{e^{-2\tau}}{-2} \bigg|_0^t = 80(e^{-2t} - 1) \text{ J}$$

$$w(1) = -69.2 \text{ J} \quad (\text{negative sign means energy supplied to the charges})$$

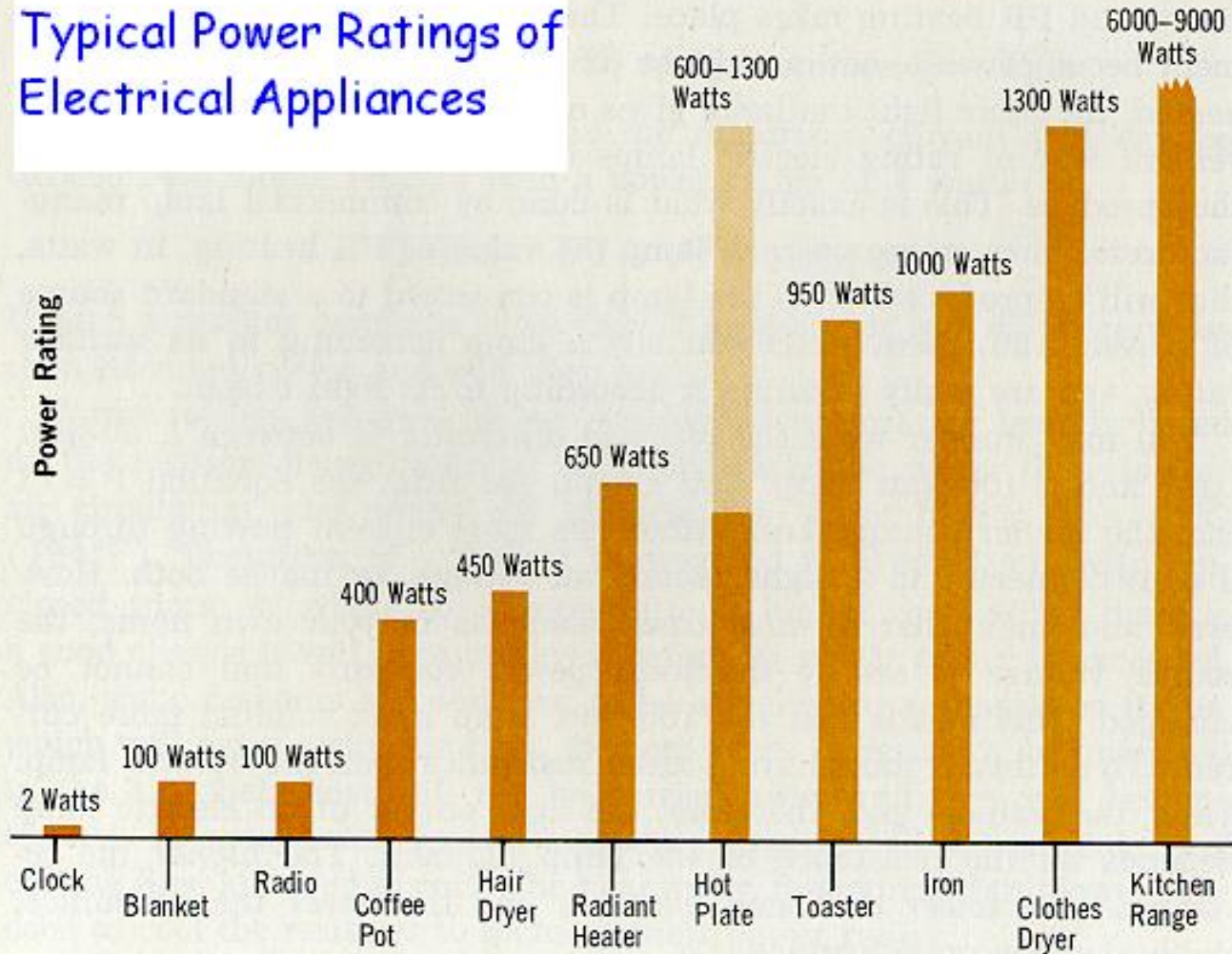
So, we say that the element is supplying 69.2 Joules of energy to the rest of the circuit over the period $0 \leq t \leq 1 \text{ sec}$.

Power/Energy: Example (cont.)

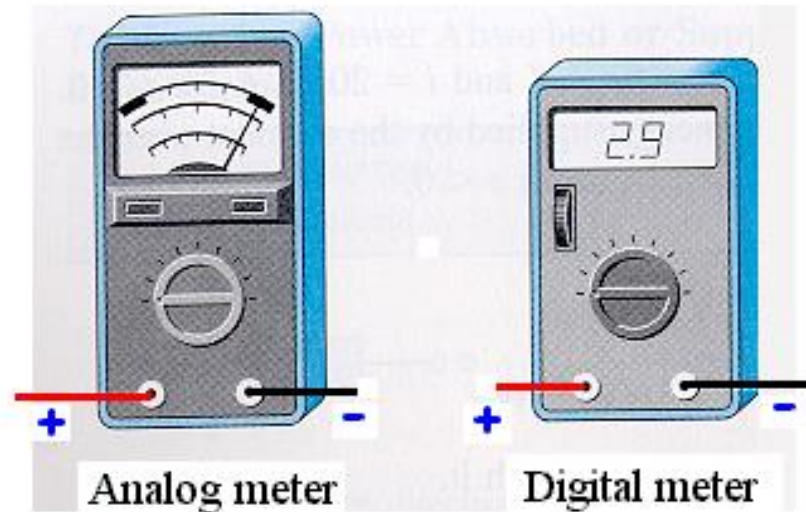
Mathcad Solution



Typical Power Ratings of Electrical Appliances



Voltmeters and Ammeters

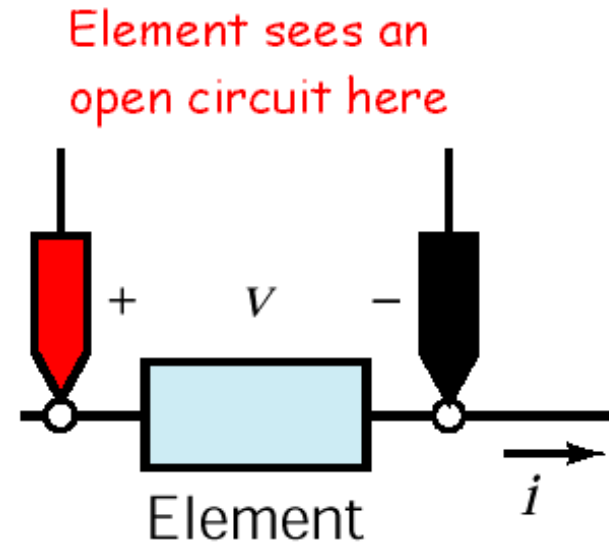
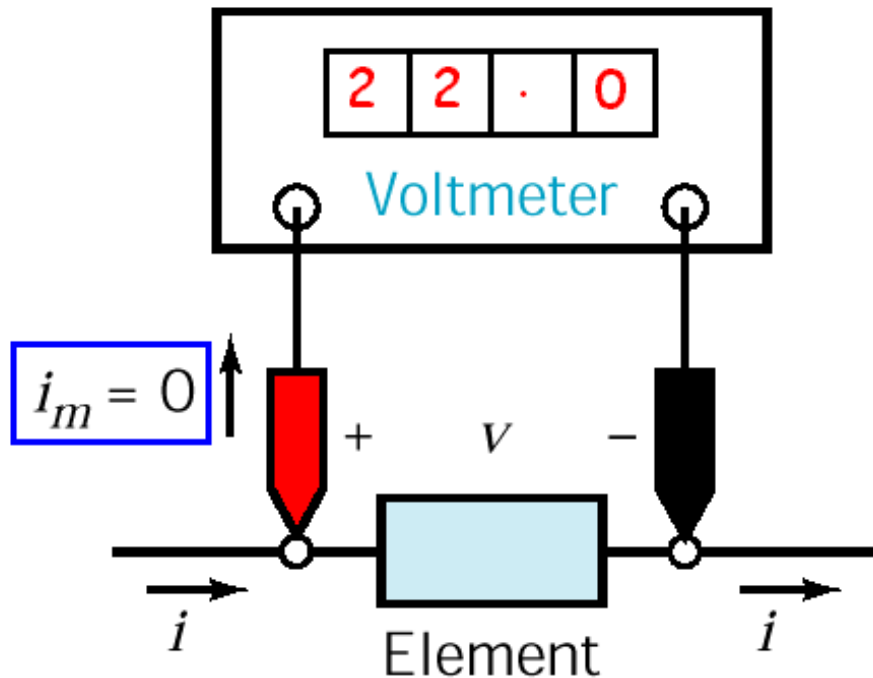


An **ammeter** is an instrument used to measure current. It has zero voltage across its terminals; when used in a circuit, it is seen as a short circuit (i.e, it acts like a conducting wire).

A **voltmeter** is an instrument used to measure voltage across an element in a circuit. It has a terminal current equal to zero. It acts as an open circuit.

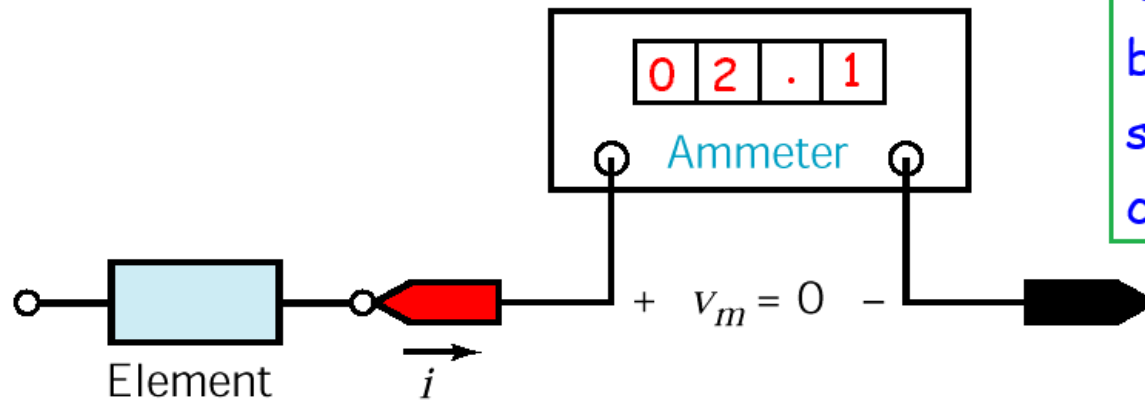
Measuring Voltage

The voltmeter must be connected in parallel to the circuit element

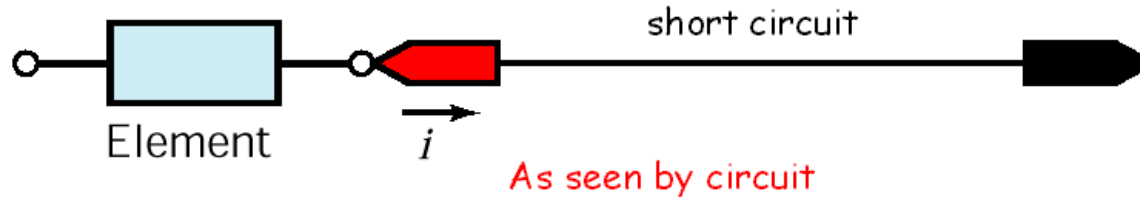


The voltmeter instrument is designed such that it does not alter the behavior of the circuit that it is connected to when it is properly placed in the circuit.

Measuring Current



The ammeter must be connected in series with the circuit element



As seen by circuit

The ammeter (Ampere meter) instrument is designed such that it does not alter the behavior of the circuit that it is connected to when it is properly placed in the circuit.

The voltmeter reads 6V which is the battery voltage AND the light bulb voltage

Positive reading means that current flows clockwise

Ammeter

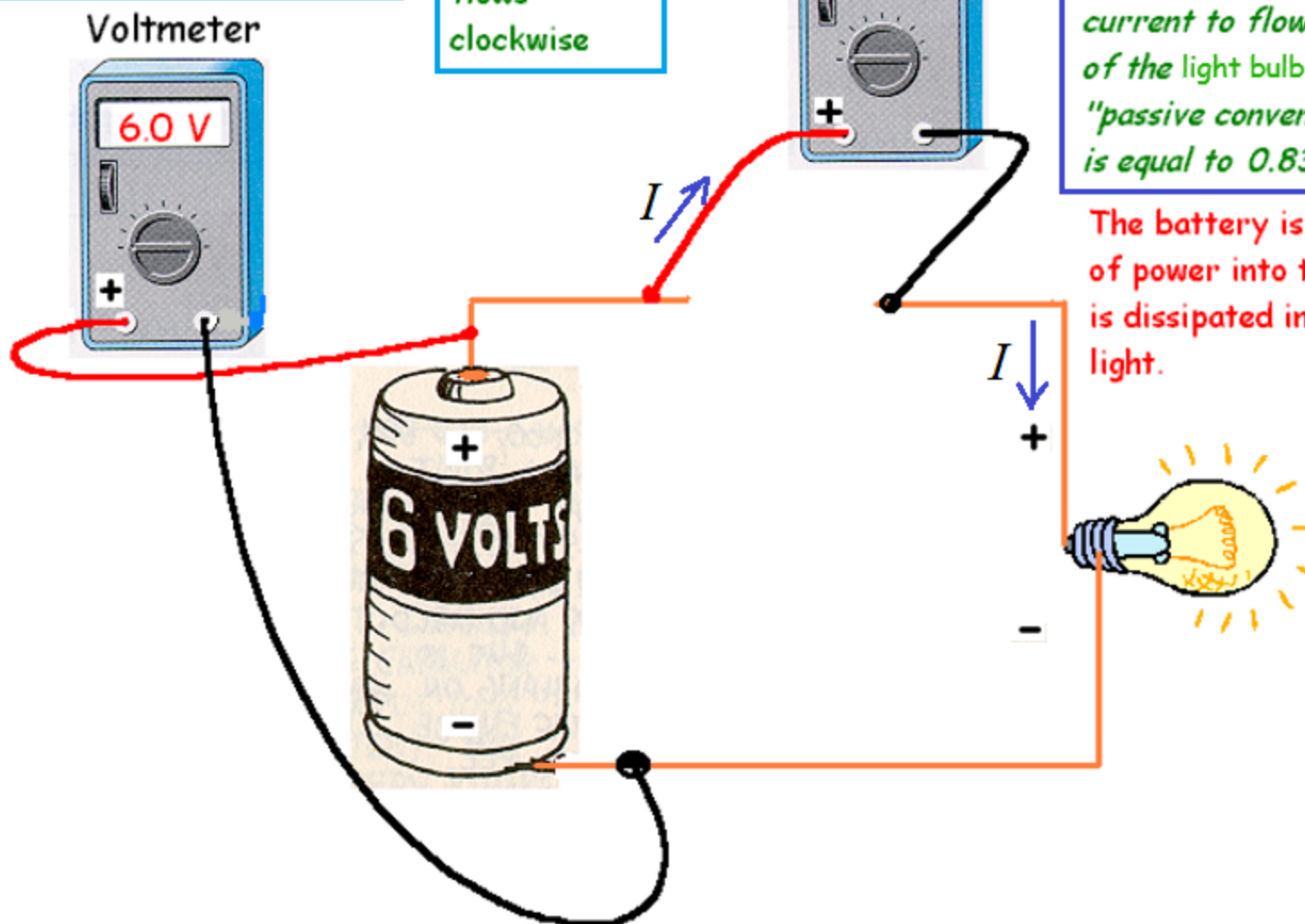
833 mA

Power calculations

light bulb : $p = 6 \times (0.833) = 4.998 \text{ W}$

Notice that we assigned the current to flow into the + terminal of the light bulb to satisfy the "passive convention." This current is equal to 0.833 A.

The battery is supplying about 5 W of power into the circuit. This power is dissipated in the lamp as heat and light.



Example:

Modern technology has produced a small 1.5 V alkaline battery with a nominal stored energy of 150 joules. For how many days will it power a calculator that draws a 2 mA current? Can you see why automatic shutoff is a good idea?

Solution:

$$p = vi = (2 \text{ mA})(1.5 \text{ V}) = 3 \text{ mW}$$

$$\Delta t = \frac{\Delta w}{p} = \frac{150 \text{ J}}{0.003 \text{ W}} = 5 \times 10^4 \text{ s}$$

$$\# \text{ of days} = (5 \times 10^4 \text{ s}) \left(\frac{1 \text{ hr}}{3600 \text{ s}} \right) \left(\frac{1 \text{ day}}{24 \text{ hr}} \right) = \underline{0.58 \text{ day}}$$

= About 14 hrs

Link: How much energy is
stored in a fresh battery?

Interpreting the Electric Utility Account Statement

Energy & Distribution Charges: 964 KWH @ 0.1609 \$154.24
--

The above boxed statement is read off a residential DTE utility statement of charges. It means that the customer is being billed \$154.24 for using 964 Kilo Watt-hour worth of energy at a rate of about 16 cents per 1 kWatt-hour of energy.

For example, if you use a 100 Watt light bulb for 10 hours, you have used a 1 kWatt-hour of energy. This will cost you 16 cents. In Joules, the energy consumed by the light bulb is:

$$1 \text{ kWh} = 1000 \left(\frac{\text{J}}{\text{s}} \right) (3600 \text{ s}) = 3.6 \text{ MJ}$$

For comparison purposes, a 9Volt battery (like the one you use in a smoke detector) stores 18,800 Joules. This means that to supply 1 kWh of energy you need about $3,600,000/18,800 \cong 191$ of the 9 Volt batteries.

Summary of Formulas

Current:

$$i(t) = \frac{dq(t)}{dt}$$

Charge:

$$q(t) = \int_{t_0}^t i(\tau) d\tau + q(t_0)$$

Instantaneous power:

$$p(t) = \frac{dw(t)}{dt} = v(t)i(t)$$

Average Power:

$$P = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} p(\tau) d\tau$$

$$P = \frac{\Delta w}{\Delta t} = \frac{w(t_1) - w(t_0)}{t_1 - t_0}$$

Average power for a periodic signal of period T :

$$P = \frac{1}{T} \int_0^T p(t) dt$$

Voltage:

$$v(t) = \frac{dw(t)}{dq}$$

$$V = \frac{W}{Q} \text{ (Total Work / Total Charge)}$$

Charge when the current is constant $i(t) = I$:

$$Q = (I)\Delta t = I(t - t_0)$$

Energy:

$$w(t) = \int_{t_0}^t p(\tau) d\tau + w(t_0)$$

Net Energy:

$$\Delta w = w(t_1) - w(t_0)$$

$$P = \frac{\Delta w}{\Delta t} = \frac{w(T) - w(0)}{T}$$

Sign interpretation in the context of electric circuits:

Negative current means that the actual direction of the current is opposite to the one you have assigned.

Assuming the "passive sign convention" we interpret the sign of power and energy in a circuit element as follows:

Positive sign means dissipated/consumed by the element.

Negative sign means generated/delivered by the element.

Refer to the next slide for the physical interpretation of the initial quantities $q(t_0)$ and $w(t_0)$.

What Does $q(t_0)$ and $w(t_0)$ Represent?

$q(t_0)$ and $w(t_0)$ are the initial values of the corresponding physical quantity. They *represent the accumulation of* charge (and energy) from “the beginning of time” (or more realistically from “the instant the physical system is put into operation”) until the time t_0 . In electric circuit analysis t_0 represents a reference time and it is usually the time a circuit is switched “ON”. Typically, we set $t_0 = 0$ as our reference time.

Formally, we define the initial value $q(t_0)$ [as we will see later, we will also be dealing with initial current $i(t_0)$ and initial voltage $v(t_0)$] for charge as

$$q(t_0) = \int_{-\infty}^{t_0} i(\tau) d\tau$$

We can determine $q(t_0)$ if we know $i(t)$ for $t < t_0$. For instance if we are told that the current is zero for $t < 0$ then we get

$$q(t_0) = \int_{-\infty}^{t_0} (0) d\tau = 0$$

Appendix

- What is an Electric Circuit?
- Requirements of a Circuit
- Electric Current
- Journey of a Typical Electron
- Power: Putting Charges to Work
- Common Misconceptions Regarding Electric Circuits
- Electric Field and the Movement of Charge
- Electric Potential
- Electric Potential Difference

Source:
physicsclassroom.com

