

Chapter 3

Resistive Circuits

Preview

The resistor (R) is an electric element commonly used in most electric circuits.

In this chapter we consider the analysis of pure resistive circuits: Circuits containing only resistors and sources.

Three laws are utilized in the analysis of resistive circuits:

1. Ohm's law ($V = RI$).
2. Kirchhoff's current law at a circuit node.
3. Kirchhoff's sum of voltages (around a closed path) law.

The Lumped element model

It is very important to note that the analysis methods considered in this introductory course (and other electronic circuit courses in the undergraduate EE program) assume that the electric circuit can be modeled accurately as a connection of discrete lumped elements. The lumped element model of electric circuits makes the simplifying assumption that the attributes of the circuit, resistance, capacitance and inductance are concentrated into idealized resistors, capacitors, and inductors, joined by a network of perfectly conducting wires.

The Lumped element model (cont.)

The lumped element model is valid as long as the time T it takes the current to go around the circuit is much smaller than the period, T_o , of the circuit's driving source, say $T < \frac{T_o}{100}$. Otherwise, when the circuit length is on the order of the source period, we must consider more general models such as the distributed element model (including transmission lines) whose dynamic behavior is described by Maxwell's equations. For example, if a sinusoidal voltage (or current) source $\cos(2\pi ft)$ drives the circuit then (for the analysis methods used in this course to be valid) we must have $T < \frac{T_o}{100} = \frac{1}{100f}$. Which means that if the current [propagating at a fraction of the speed of light, say $0.7c \cong (2)10^8 \text{ meter/sec}$] goes around a closed circuit path of length L we must have $\frac{L}{0.7c} < \frac{1}{100f}$ or $f < \frac{0.7c}{100L}$.

For example, if we have a circuit build on a breadboard with longest path $L = 1$ meter then the lumped model would require that the source frequency (in Hz) be smaller than $f < \frac{0.7c}{100L} = \frac{(2)10^9}{100} = 20\text{MHz}$.

Many useful discrete-component circuits operate at frequencies that are much less than 20MHz. Also, if the circuit dimension is decreased (think integrated circuits) then the lumped model would still work for frequencies of the order of GHz (think clock frequency inside your modern laptop PC).

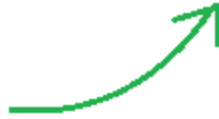
The Basic Circuit Analysis Problem

This course is about the analysis of ideal, linear, time-invariant, lumped electric circuits. It is analysis vs the more difficult synthesis (design) problem. It is ideal element models vs the much more complicated physical models. Linear and time-invariant (as opposed to non-linear and time-variant) circuits allow us to model such circuits with linear, time-invariant coefficient ordinary differential equations. The exact analytical solution of such equations is relatively easy to determine. [The non-linear varying-coefficient differential equation generally does not have exact analytical solutions and requires numerical methods for its solution.] Finally, the lumped circuit allows us to obtain the circuit mathematical model by applying Kirchhoff's Laws (and the $v - i$ element relationships). If the lumped-element model does not apply, then our analysis problem becomes a much more difficult problem of “distributed circuit” analysis that requires the use of Maxwell's equations.

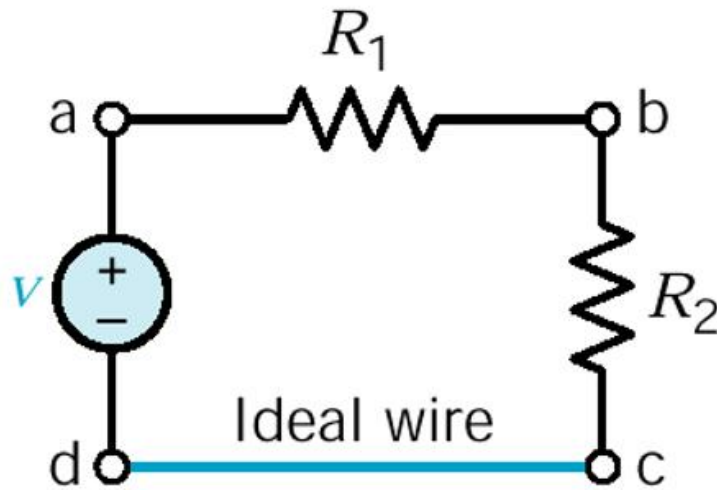
The restriction to the analysis of ideal, linear, time-invariant, lumped circuits might appear to be a severe limitation. However, many important problems fall into such a classification. Furthermore, the understanding of the material in this course is essential when such restrictions are removed in more advanced courses.

A Brief History of Electric Circuit Analysis

LINK

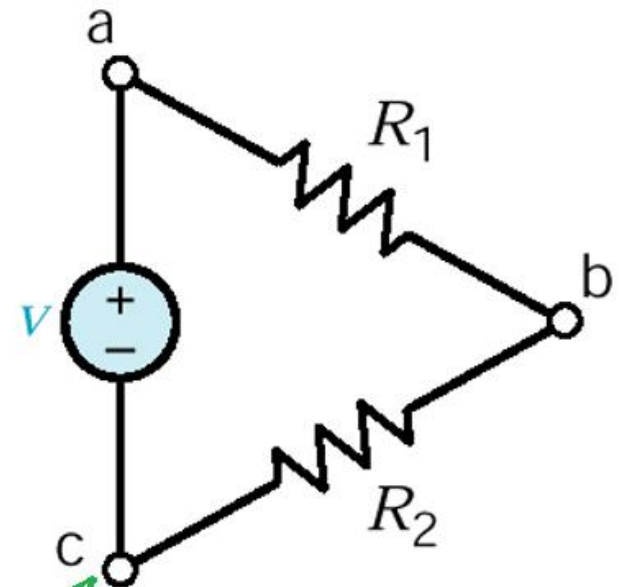


A Simple 2-Resistor Circuit with one Independent Voltage Source



a, b, c, and d are called nodes.

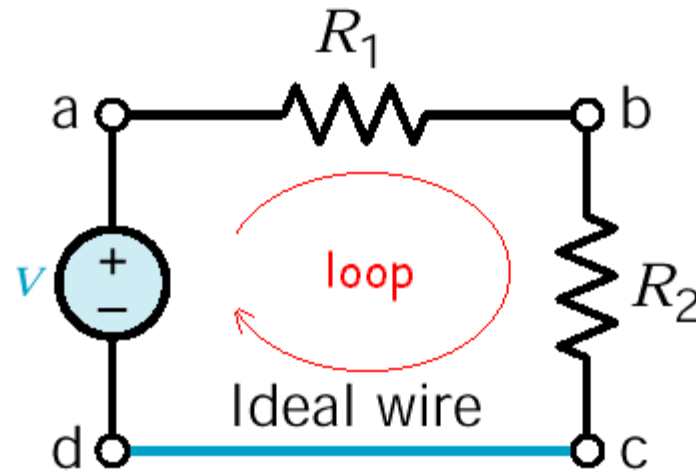
OR



We may collapse a "wire" into a single node

A *node* is a junction in which two or more elements have a common connection. This junction is made of ideal conducting wire (i.e., has a zero resistance).

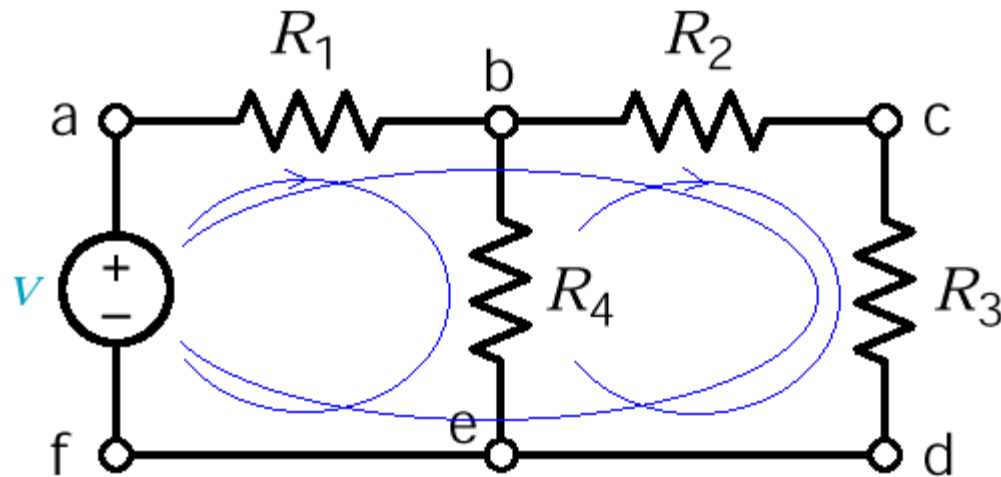
A Simple 2-Resistor Circuit with one Independent Voltage Source



If we start at node a and traverse around the circuit, passing through each node in turn (i.e., a to b to c to d, and then back to a), then we have traversed a *closed path*.

A closed path is often called a *loop*.

Example: Identifying Closed Paths (loops) in a circuit



There are three closed paths (loops) in this circuit:

Loop 1: a-b-e-f-a

Loop 2: b-c-d-e-b

Loop 3: a-b-c-d-e-f-a

Note that we could also refer to, say, loop 1 as: a-f-e-b-a. Normally, we choose clock-wise direction.

The Law of Conservation of Charge

Electric charge can neither be created nor destroyed. The net quantity of electric charge in a closed system (electric circuit, universe, etc.) is always conserved.

Charge conservation is a physical law that states that the change in the amount of electric charge in any volume of space is exactly equal to the amount of charge flowing into the volume minus the amount of charge flowing out of the volume. Mathematically, we can state the law as a continuity equation:

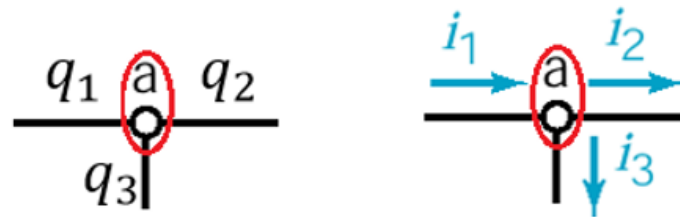
$$Q(t_2) = Q(t_1) + Q_{in} - Q_{out}$$

$Q(t)$ is the quantity of electric charge in a specific volume at time t , Q_{in} is the amount of charge flowing into the volume between time t_1 and t_2 , and Q_{out} is the amount of charge flowing out of the volume during the same time period.

KCL and the Law of Conservation of Charge

No charge can be stored in a circuit node. Therefore, by the law of conservation of charge, the amount of charge (at time t) in a wire connected to a node must be equal to the sum of charges in all remaining wires connected to that same node (at time t). For the portion of a circuit shown below we can write

$$q_1(t) = q_2(t) + q_3(t)$$



If a net charge moves into the node (causing an electric current $\frac{dq}{dt}$) that will cause currents to flow in all wires connected to the node. These currents are governed by the derivative of Eq. (1) leading to KCL

$$\frac{dq_1}{dt} = \frac{dq_2}{dt} + \frac{dq_3}{dt} \rightarrow i_1 = i_2 + i_3$$

Kirchhoff's Laws



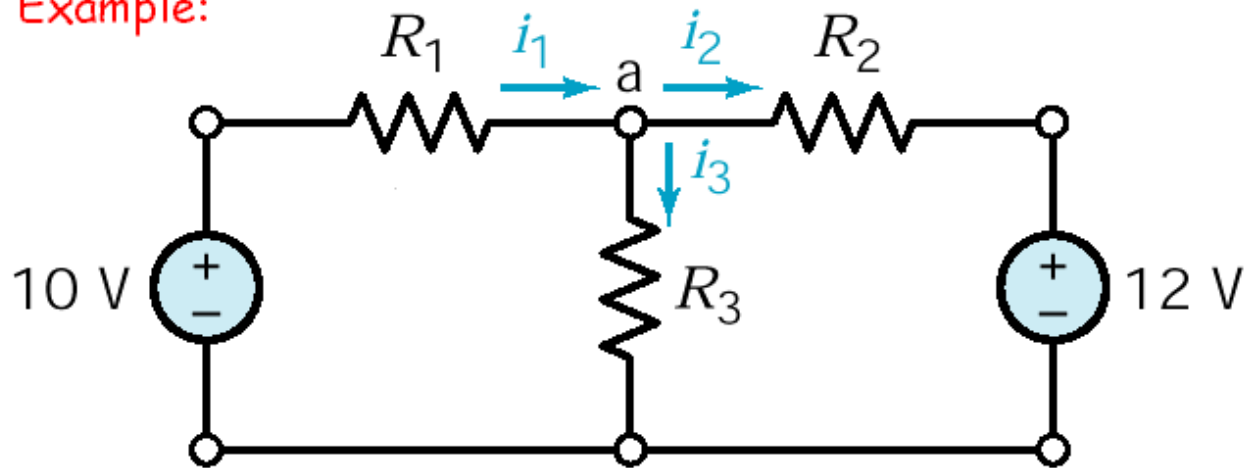
Gustav Kirchhoff (1824-1887), Professor at the University of Berlin, formulated in 1847 two laws that relate the current and voltage in a circuit with two or more resistors.

Note: Ohm's law only relates the current of a given resistor to its voltage.

Kirchhoff's Laws

Kirchhoff's current law (KCL): the algebraic sum of the currents entering any node is identically zero for all instants of time.

Example:



$$i_1 - i_2 - i_3 = 0$$

or $i_1 = i_2 + i_3$

Kirchhoff's current law is a direct consequence of the law of conservation of charge

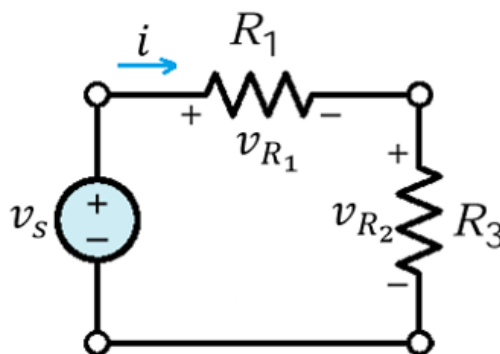
(sum of currents flowing into a node) = (sum of currents leaving node)

The Law of Conservation of Energy

The **law of conservation of energy** states that the total energy of an isolated system remains constant—it is said to be *conserved* over time. Energy can neither be created nor destroyed; rather, it transforms from one form to another.

KVL and the Law of Conservation of Energy

Consider the single loop circuit consisting of two resistors in series driven by a voltage source (the resistors can be replaced by other circuit elements: capacitors, resistors, etc.)



By the law of conservation of energy (or power), the power delivered by the source at time t , $p_s(t)$, must be equal to the power dissipated (as heat) in the resistors, $p_{R_1}(t)$ and $p_{R_2}(t)$ [Note the proper use of the *passive sign convention*]:

$$p_s(t) = p_{R_1}(t) + p_{R_2}(t) \quad \rightarrow \quad v_s i = v_{R_1} i + v_{R_2} i$$

The current i cancels out leading to KVL:

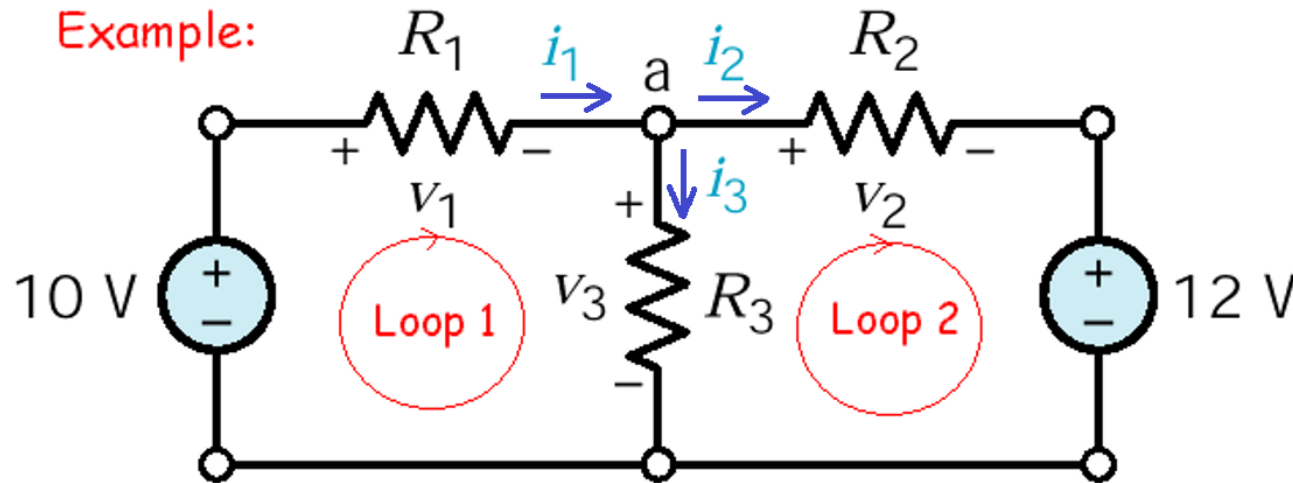
$$v_s = v_{R_1} + v_{R_2} \quad \text{or} \quad -v_s + v_{R_1} + v_{R_2} = 0$$

Kirchhoff's Laws

Kirchhoff's voltage law is a direct consequence of the law of conservation of energy

Kirchhoff's voltage law (KVL): the algebraic sum of the voltages around any closed path in a circuit is identically zero for all time.

Example:



$$\begin{array}{ll} \text{Loop 1:} & \text{Loop 2:} \\ -10 + v_1 + v_3 = 0 & -v_3 + v_2 + 12 = 0 \end{array}$$

$$\text{Loop 3 (outer loop): } -10 + v_1 + v_2 + 12 = 0$$

Example (cont.)

KVL:

$$\text{Loop 1: } -10 + v_1 + v_3 = 0 \quad (1)$$

$$\text{Loop 2: } -v_3 + v_2 + 12 = 0 \quad (2)$$

$$\text{Loop 3 (outer loop): } -10 + v_1 + v_2 + 12 = 0 \quad (3)$$

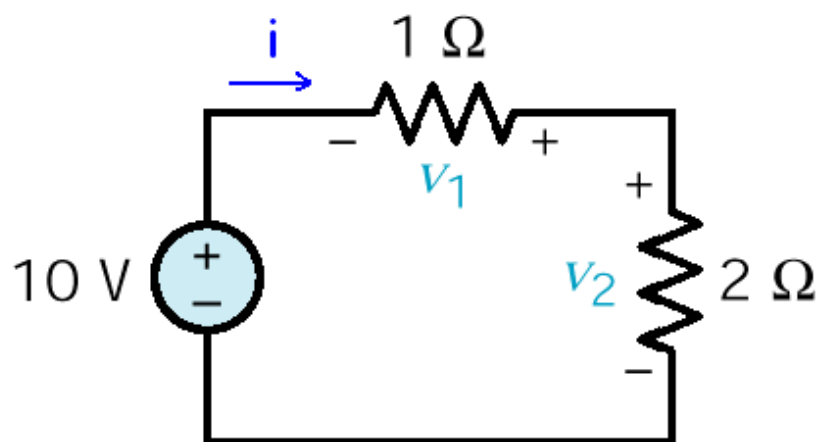
Note that Equation (3) is linearly dependent on (can be derived from) Eqn. (1) and (2), so we ignore it. We have three unknowns and two equations. So, we need a third equation in the voltage variables. This can be obtained by applying KCL at node a :

$$i_1 - i_2 - i_3 = 0 \rightarrow \frac{v_1}{R_1} - \frac{v_2}{R_2} - \frac{v_3}{R_3} = 0 \quad (4)$$

Given numerical values for the resistors, Equations (1), (2) and (4) can be solved for the voltages v_1 , v_2 and v_3 .

Example

Find the current in the following circuit:



Employing KVL, we get:

$$-10 - v_1 + v_2 = 0 \quad \text{..... (1)}$$

We may relate v_1 and v_2 to i using Ohm's law at each resistor:

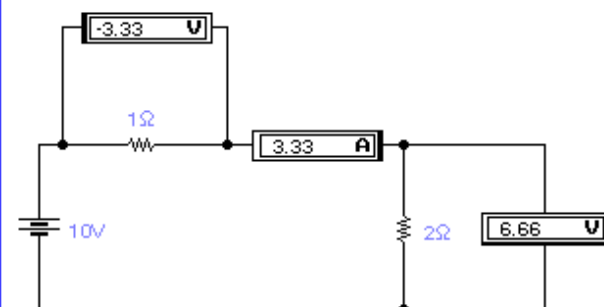
$$v_1 = (1)(-i) = -i \quad \text{and} \quad v_2 = (2)i = 2i$$

which upon substituting in Eq. (1) gives:

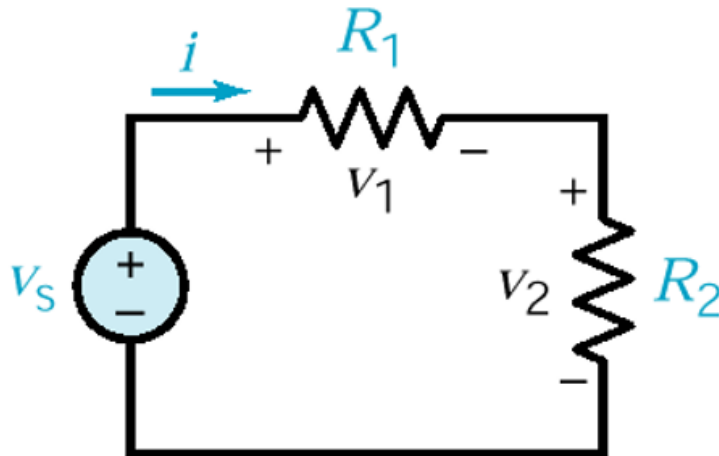
$$-10 - (-i) + 2i = 0 \quad \text{or} \quad i = 10/3 \text{ A}$$

Electronics Workbench

Workbench Solution



The Voltage Divider Circuit



The two resistors are said to be connected in series since they both share the same current.

Applying KVL we get: $-v_s + v_1 + v_2 = 0$

Now, by Ohm's law: $v_1 = R_1 i$ and $v_2 = R_2 i$

By combining the above equations we get:

$$-v_s + R_1 i + R_2 i = 0 \quad \text{or} \quad i = \frac{v_s}{R_1 + R_2}$$

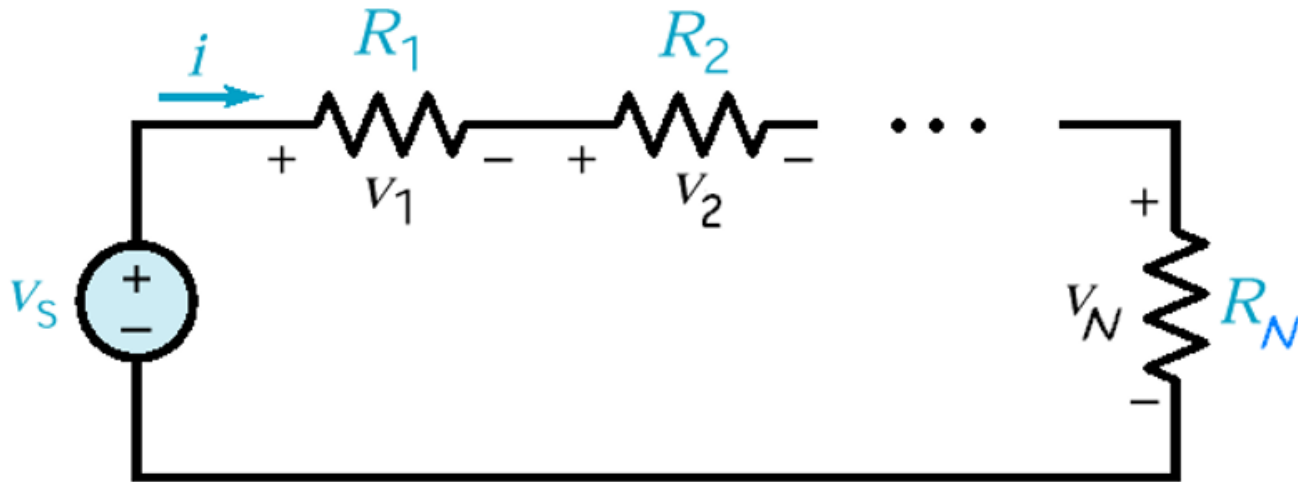
Voltage division formulas

Finally, we may express v_1 & v_2 as:

$$v_1 = R_1 i = \frac{R_1}{R_1 + R_2} v_s$$

$$v_2 = R_2 i = \frac{R_2}{R_1 + R_2} v_s$$

General form of The Voltage Divider Circuit

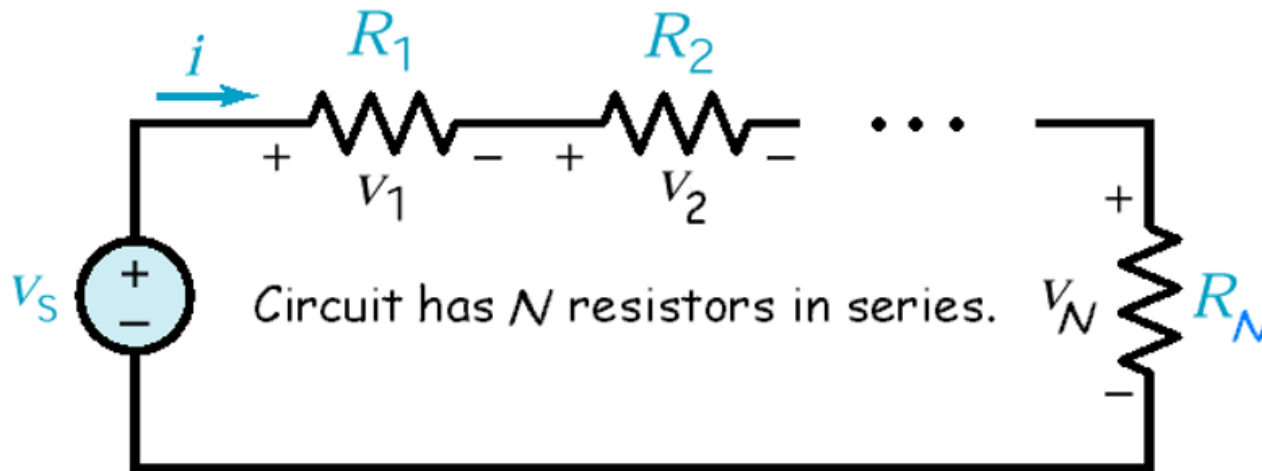


Circuit has N resistors in series.

The voltage across the n th resistor is given by:

$$v_n = \frac{R_n}{R_1 + R_2 + \dots + R_N} v_s$$

Equivalent Resistance



KVL: $-v_s + v_1 + v_2 + \dots + v_N = 0$

$$-v_s + R_1 i + R_2 i + \dots + R_N i = 0 \quad (\text{using Ohm's law})$$

Solving for i :

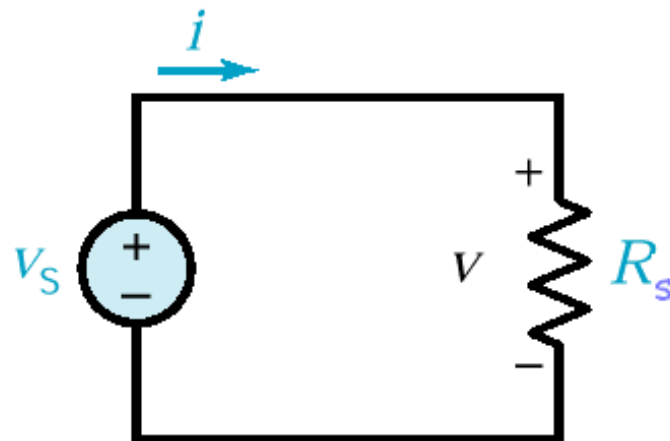
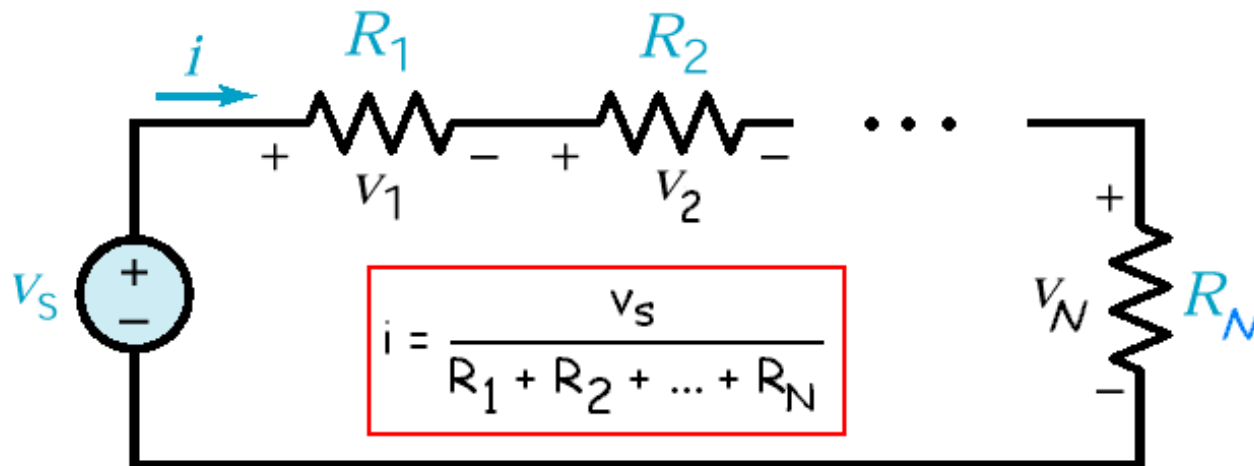
$$i = \frac{v_s}{R_1 + R_2 + \dots + R_N} = \frac{v_s}{R_s}$$

Where

$R_s = R_1 + R_2 + \dots + R_N$ is the *equivalent resistance*.

Resistors connected in series (will share the same current) and can be replaced by a single resistor equal to their sum.

Equivalent Resistance



KVL: $-v_s + v = 0$
 or $-v_s + R_s i = 0$

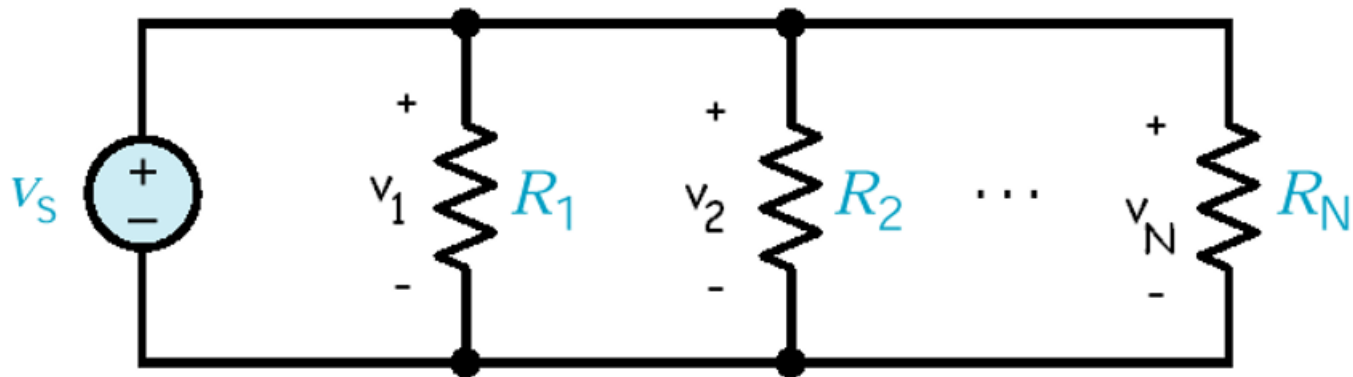
$i = \frac{v_s}{R_s}$

The two circuits are equivalent if $R_s = R_1 + R_2 + \dots + R_N$

By "equivalent" we mean that the voltage source "sees" the same load.
 i.e., drives the same current or delivers the same power $P = V_s i = \frac{v_s^2}{R_s}$

Parallel Resistive Circuits

Circuit with a voltage source



KVL: Loop 1: $-v_S + v_1 = 0 \rightarrow v_1 = v_S$

Loop 2: $-v_S + v_2 = 0 \rightarrow v_2 = v_S$

⋮

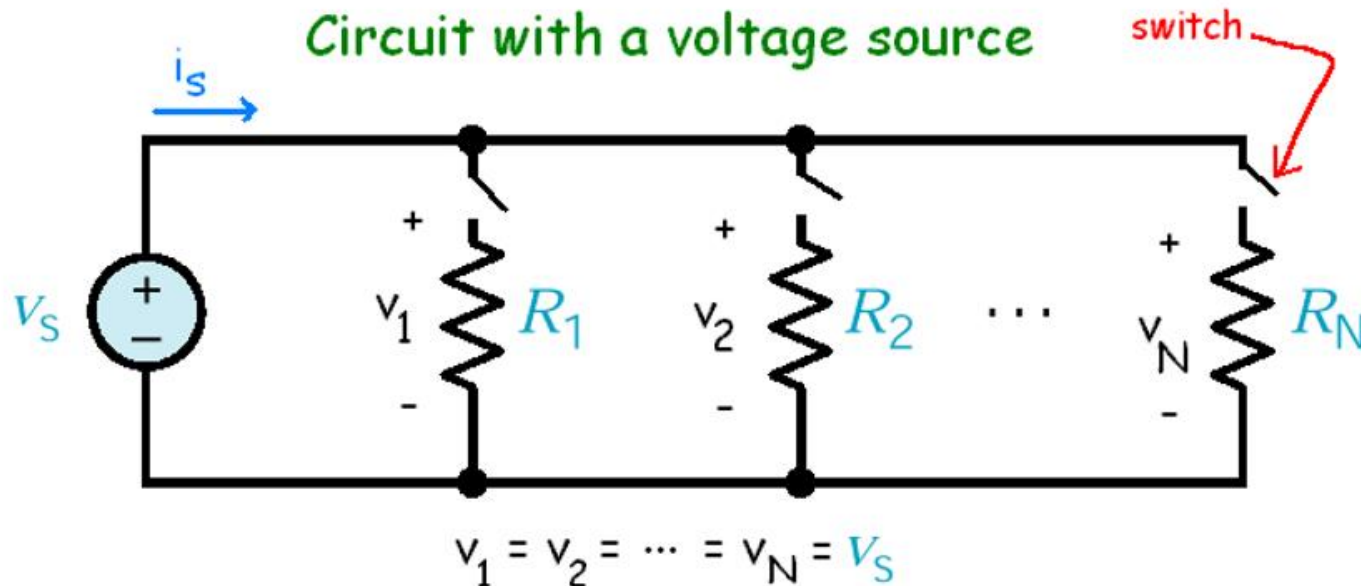
Loop N: $-v_S + v_N = 0 \rightarrow v_N = v_S$

Therefore: $v_1 = v_2 = \dots = v_N$

Resistors connected in parallel share the same voltage.

Parallel Resistive Circuits

Circuit with a voltage source



Electric house wiring conforms to the above circuit. All lamps are connected in parallel.

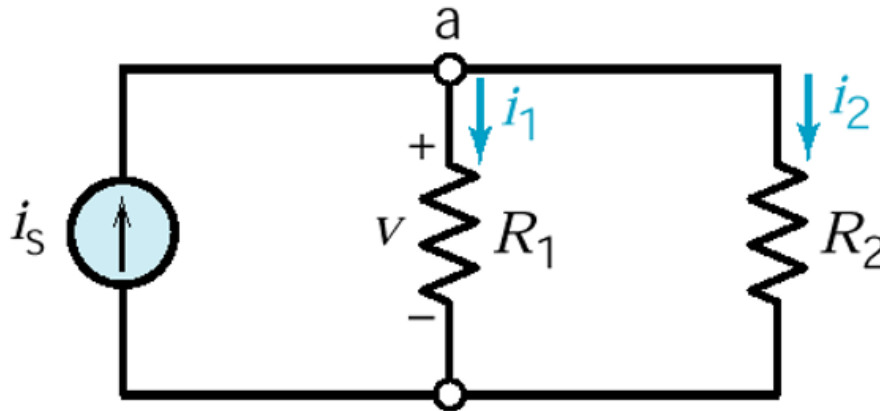
When a switch is closed, the corresponding lamp (with resistance R) receives the voltage v_S .

According to Ohm's law, the current in each lamp (or device with resistance R_n) is $i_n = v_S / R_n$.

The source must be capable of supplying $i_S = i_1 + i_2 + \dots + i_N$ Amps in the extreme case. i_S max is determined by a circuit breaker.

Parallel Resistive Circuits

Circuit with a current source



KCL at node a: $i_s = i_1 + i_2$ (1)

Since the two resistors are in parallel, then they have the same voltage. Let us call this voltage v .

From Ohm's law: $i_1 = v/R_1$ and $i_2 = v/R_2$

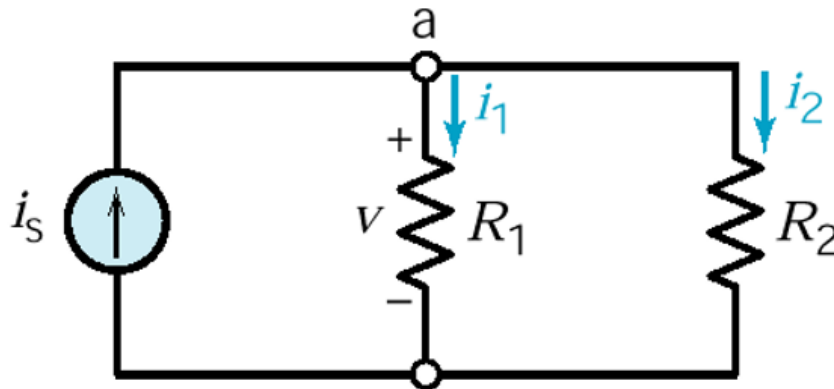
Substituting in Eq. (1):

$$i_s = \frac{v}{R_1} + \frac{v}{R_2} = \frac{R_1 + R_2}{R_1 R_2} v \quad \text{or} \quad v = \frac{R_1 R_2}{R_1 + R_2} i_s$$

The equivalent resistance of two parallel resistors R_1 and R_2 is given by: $\frac{R_1 R_2}{R_1 + R_2}$

Parallel Resistive Circuits

The Current Divider



$$v = \frac{R_1 R_2}{R_1 + R_2} i_s$$

The currents i_1 and i_2 may now be expressed as:

$$i_1 = \frac{v}{R_1} = \frac{R_2}{R_1 + R_2} i_s$$

$$i_2 = \frac{v}{R_2} = \frac{R_1}{R_1 + R_2} i_s$$

The current division formulas:

Alternatively, the currents may be expressed in terms of the conductances as follows:

$$\text{Let } G_1 = \frac{1}{R_1} \text{ and } G_2 = \frac{1}{R_2}$$

and

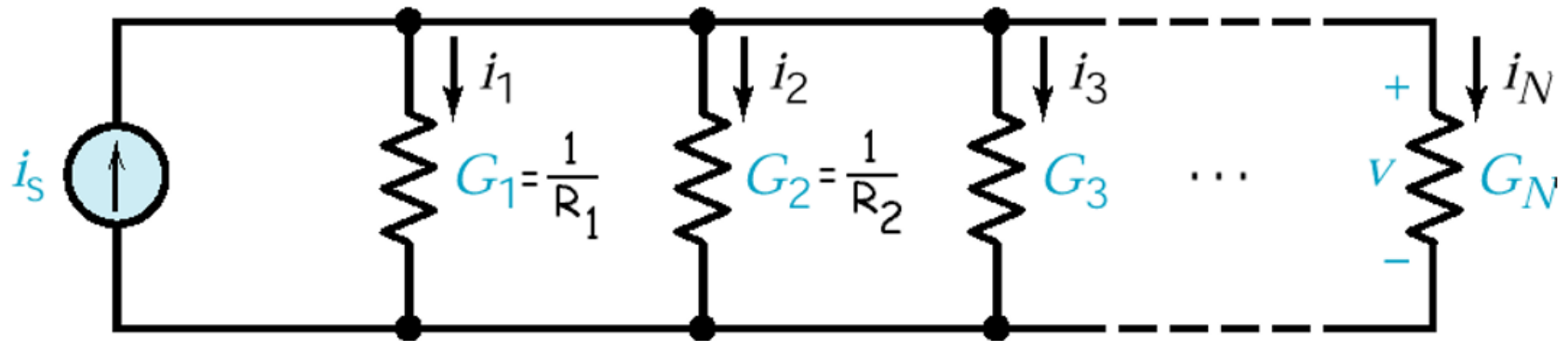
$$G_p = G_1 + G_2$$

Then:

$$i_1 = \frac{G_1}{G_p} i_s \text{ and } i_2 = \frac{G_2}{G_p} i_s$$

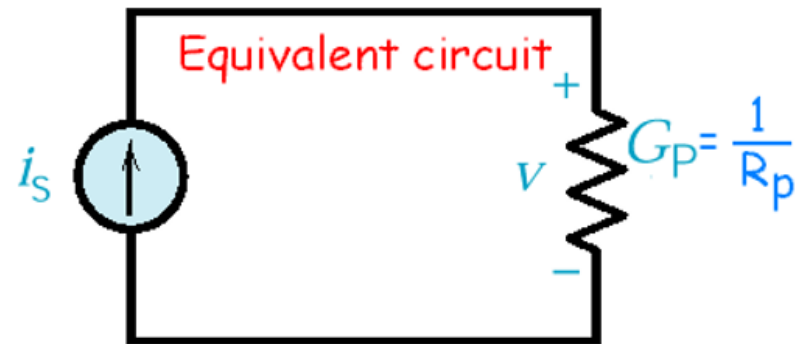
The equivalent conductance of two resistors in parallel is equal to the sum of conductances

Generalized Current Divider



For an N resistor parallel circuit, the equivalent circuit will have a resistance R_p given by:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$$



The current i_n in the n th resistor is given by:

Current division rule:

$$i_n = \frac{G_n}{G_p} i_s$$

In Summary:

Series resistors can be added up:

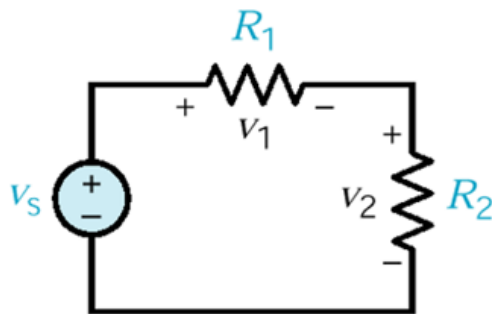
$$R_{eq} = R_1 + R_2 + \dots$$

Parallel resistors conductance can be added up:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots \quad \text{or} \quad G_{eq} = G_1 + G_2 + \dots$$

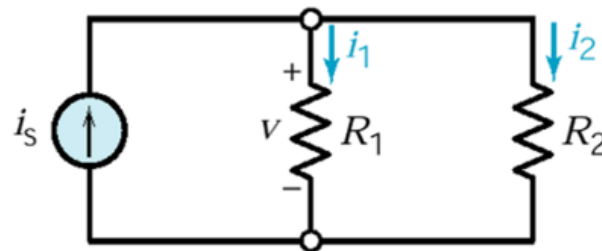
Case of two resistors in parallel: $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

Voltage division:



$$v_2 = \frac{R_2}{R_1 + R_2} v_s$$

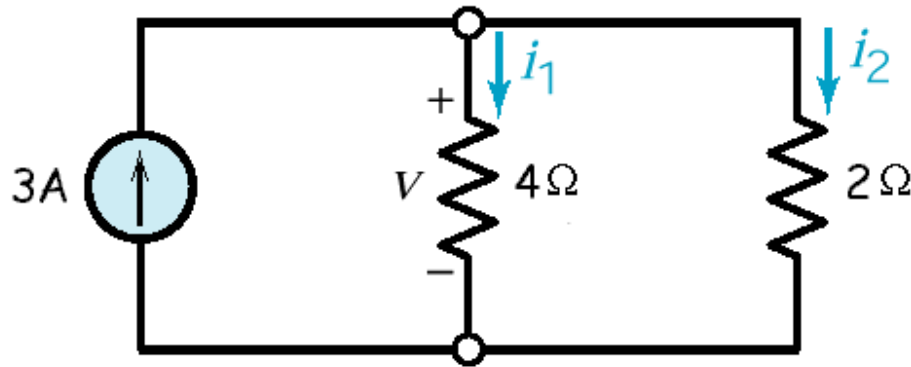
Current division:



$$i_2 = \frac{R_1}{R_1 + R_2} i_s$$

Example

Find the currents i_1 and i_2 and the voltage v in the following circuit.



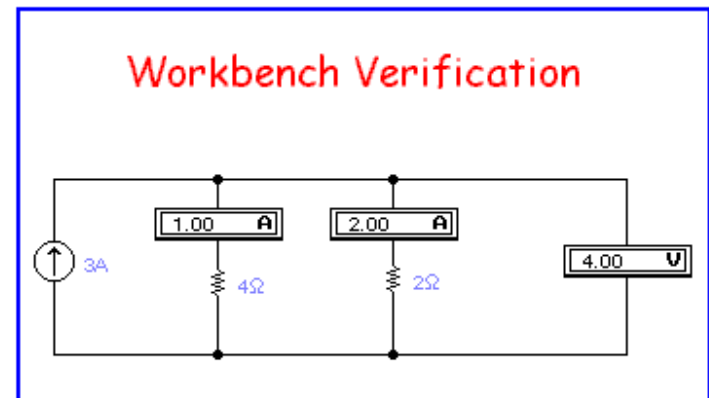
Solution:

$$i_1 = \frac{R_2}{R_1 + R_2} i_s = \frac{2}{4 + 2} 3 = 1\text{A}$$

$$i_2 = \frac{R_1}{R_1 + R_2} i_s = \frac{4}{4 + 2} 3 = 2\text{A}$$

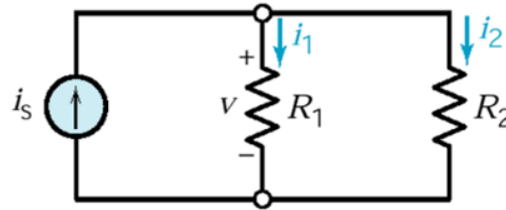
$$v = R_1 i_1 = (4)(1) = 4\text{V}$$

Electronics Workbench



Current Divider in the Limit of one Resistor Approaching Zero

Current divider:



Current “prefers” the *path of least resistance*. In a current divider, more current will flow in the lower resistance branch compared to the higher resistance branch. That can be easily verified by inspecting the current division formulas:

$$i_1 = \frac{R_2}{R_1 + R_2} i_s \quad \text{and} \quad i_2 = \frac{R_1}{R_1 + R_2} i_s$$

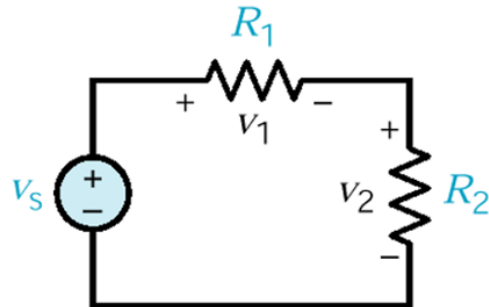
If we set $R_1 = 0$ (replace R_1 by a short circuit component) then

$$i_1 = \frac{R_2}{0 + R_2} i_s = i_s \quad \text{and} \quad i_2 = \frac{0}{0 + R_2} i_s = 0$$

Therefore, the *whole source current will flow in the zero resistance branch*.

Voltage Divider in the Limit of one Resistor Approaching Zero

Voltage divider:



For a voltage divider, more voltage will develop across the resistor with the larger resistance. And, in the extreme case where one of the resistors becomes a short circuit (say, $R_1 = 0$) then the full source voltage v_s will appear across the non-zero resistance (across R_2). This observation is sort of obvious, but it is confirmed analytically by setting $R_1 = 0$ to get

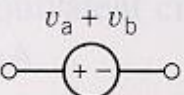
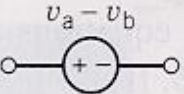
$$v_2 = \frac{R_2}{R_1 + R_2} v_s = \frac{R_2}{0 + R_2} v_s = v_s$$

and

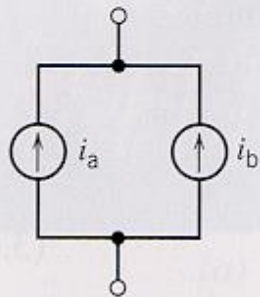
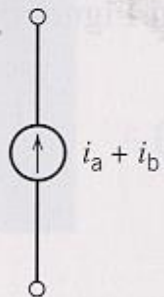
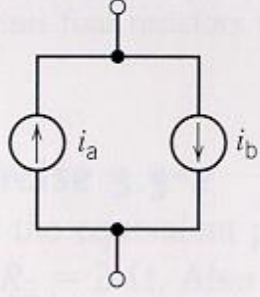
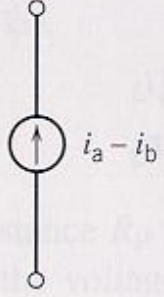
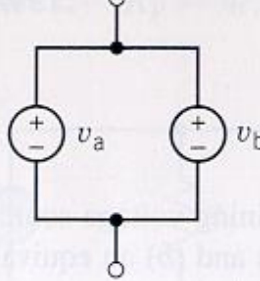
$$v_1 = \frac{R_1}{R_1 + R_2} v_s = \frac{0}{0 + R_2} v_s = 0$$

Combining Independent Voltage (Current) Sources

Sources in Series

Circuit	Equivalent Circuit
	
	
	Not allowed unless $i_a = i_b$

Sources in Parallel

Circuit	Equivalent Circuit
	
	
	Not allowed unless $v_a = v_b$

Examples of Resistive Circuit Analysis

Example 1:

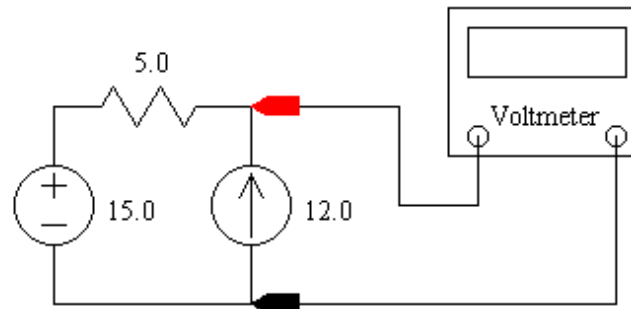
Prof Svoboda's DC Circuit Tutor.

Help

Calculator

New Circuit

Show Answer



Run DC Tutor

What is the value of the voltage measured by the voltmeter?

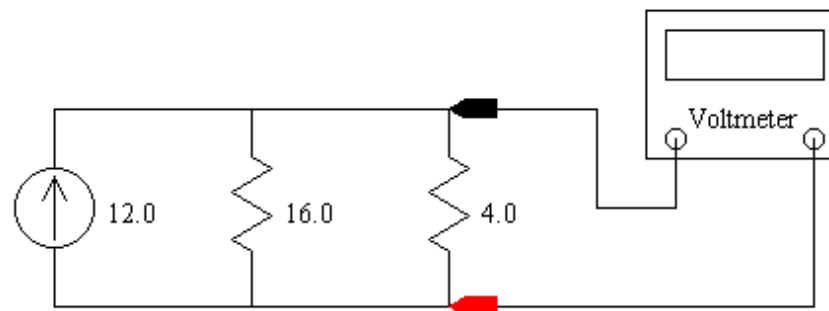
Example 2:

Help

Calculator

New Circuit

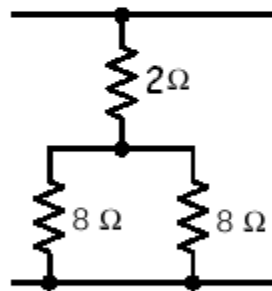
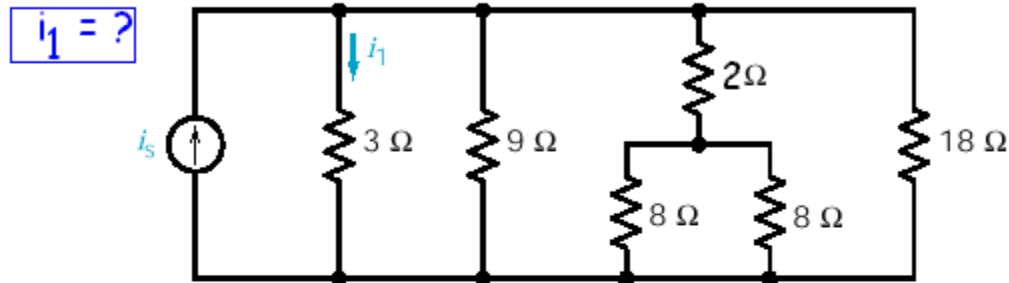
Show Answer



What is the value of the voltage measured by the voltmeter?

Examples of Resistive Circuit Analysis

(More General Examples)



simplify into -->

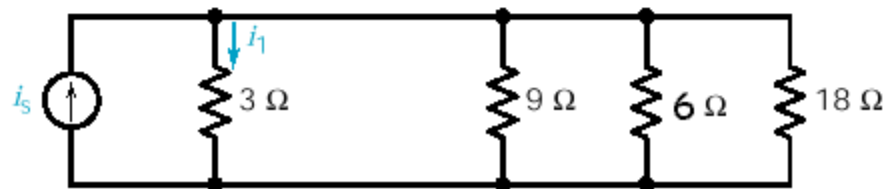


where

$$R_s = 2 + \frac{(8)(8)}{8 + 8}$$

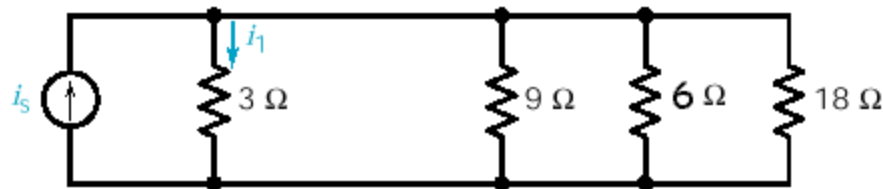
$$= 2 + 4 = 6 \Omega$$

Circuit reduces the following equivalent circuit:

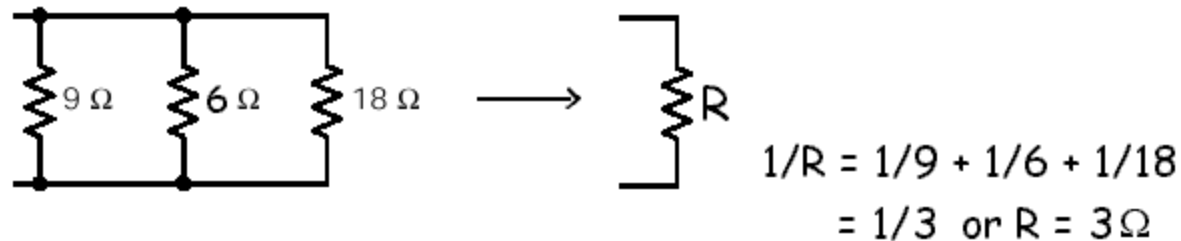


Examples of Resistive Circuit Analysis

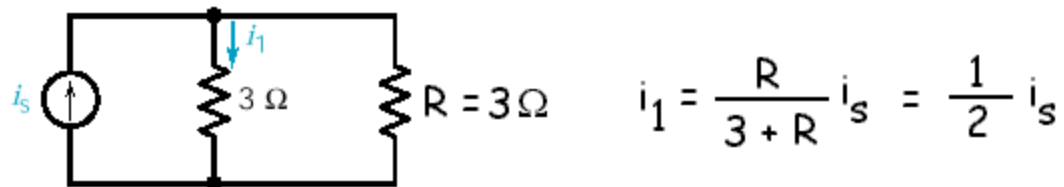
Example (cont.)



Next, the three parallel resistors reduce to:



Finally, using current division:

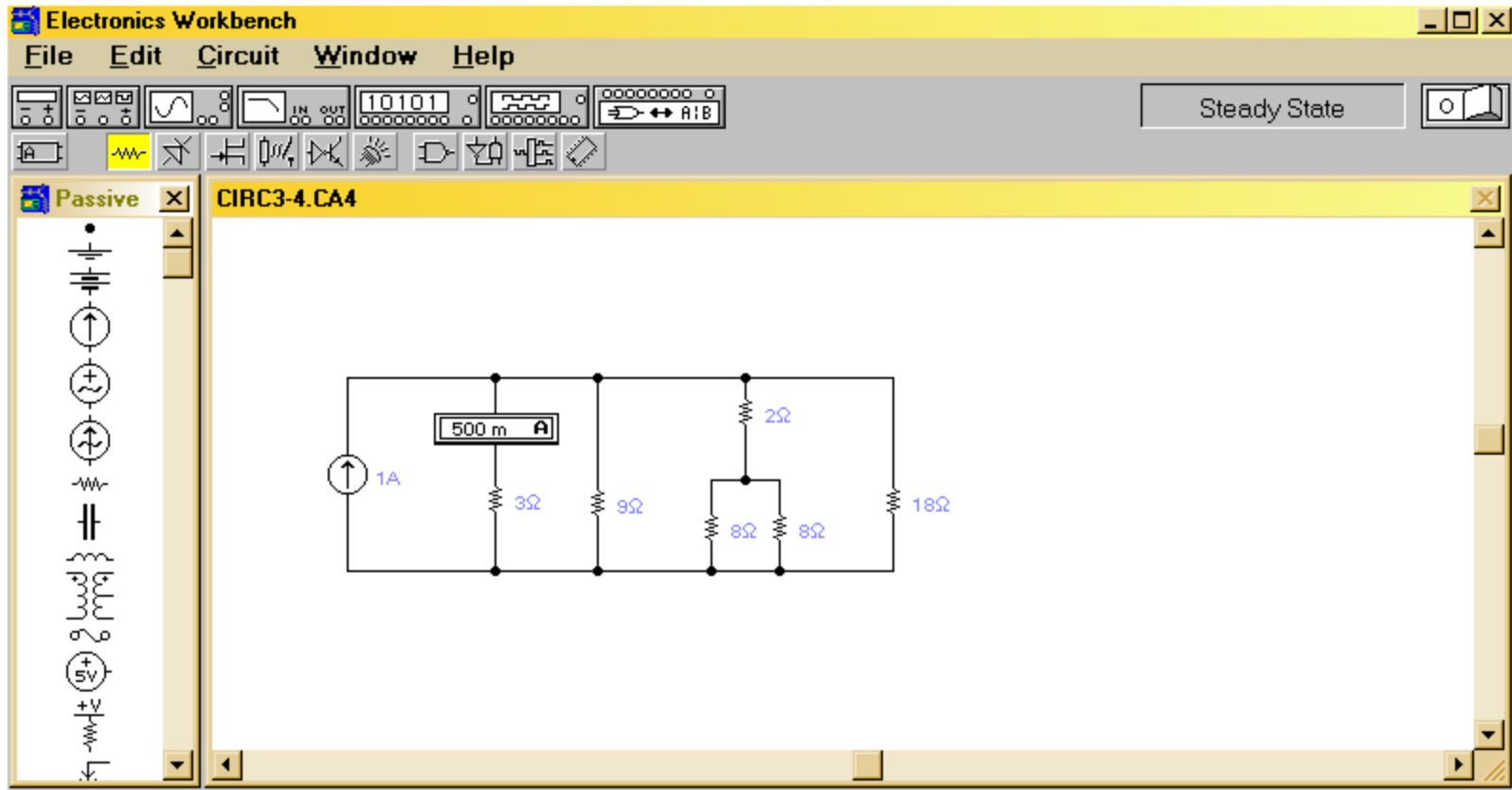


Examples of Resistive Circuit Analysis

Example (cont.)

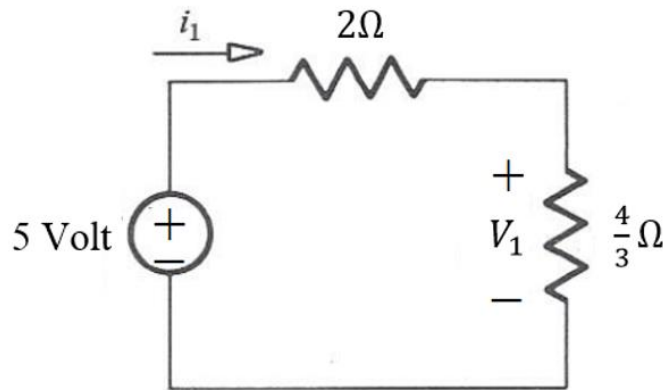
Workbench Example

Electronics Workbench



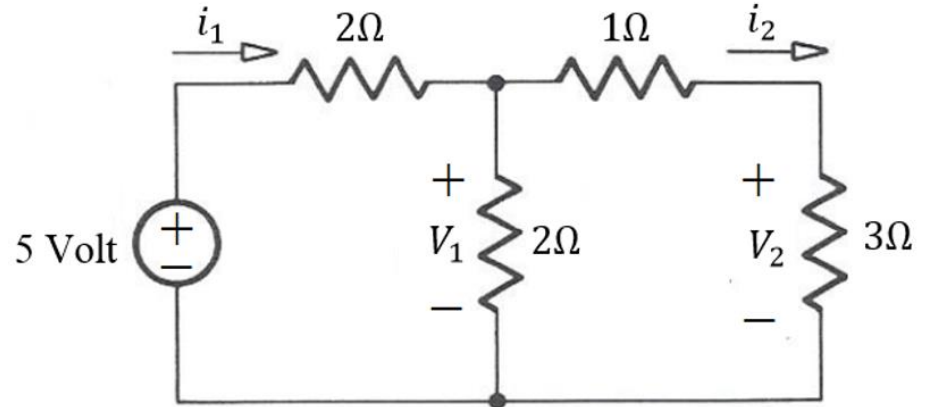
Example: Determine V_2 employing equivalent resistance and voltage division principles.

If we combine the 1Ω and 3Ω resistors in series to get 4Ω , and then combine this resistor with the 2Ω resistor (in parallel) we reduce the original circuit to the one shown below.



Next, by voltage division we get

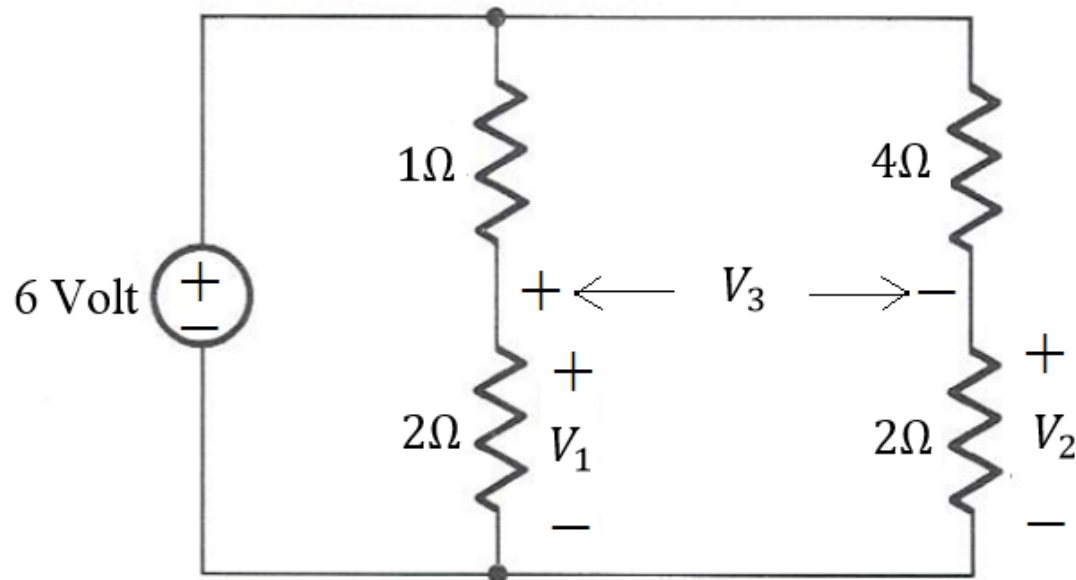
$$V_1 = \frac{\frac{4}{3}}{2 + \frac{4}{3}} 5 = 2 \text{ Volt.}$$



Returning to the above circuit and employing voltage division (while treating V_1 as the source voltage) we get the desired answer:

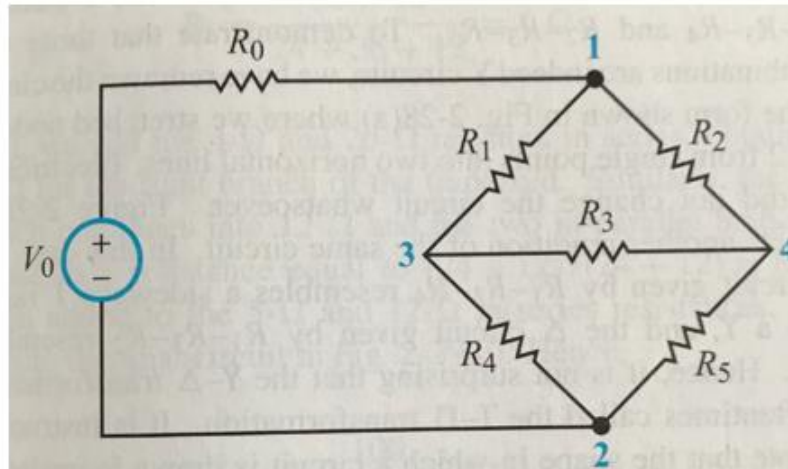
$$V_2 = \frac{3}{1+3} (2) = \frac{3}{2} \text{ Volt}$$

Your turn: Determine V_3 employing voltage division.



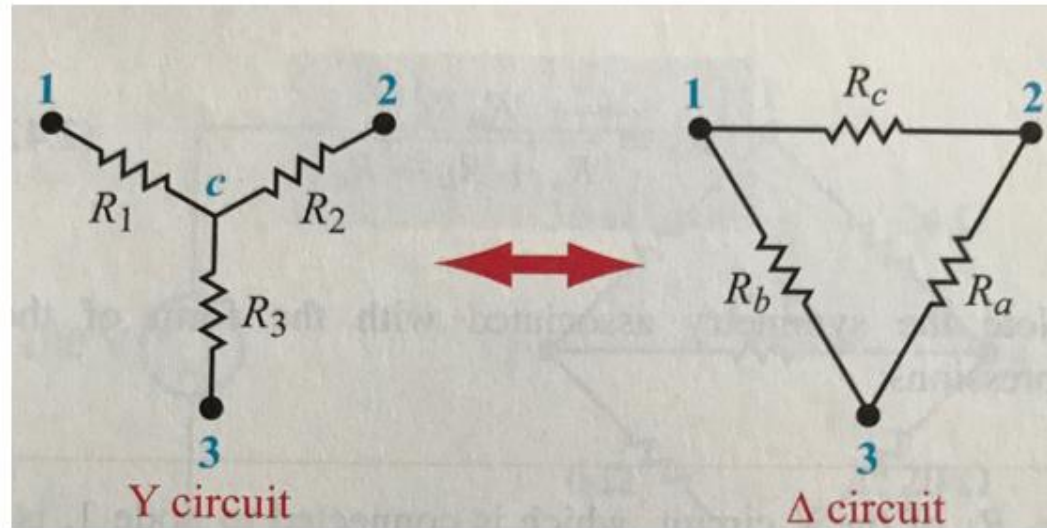
The Wye-Delta (Y – Δ) Transformation

There are situations where the resistors are connected in a circuit such that the elementary method of parallel and series combination of resistors does not apply because resistors do not share the same current (series connection) or same voltage (parallel connection). The following circuit poses such a situation.



The next slide presents a set of formulas that can be used to handle such cases.

The Wye-Delta (Y – Δ) Transformation



$\Delta \rightarrow Y$ Transformation:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$Y \rightarrow \Delta$ Transformation:

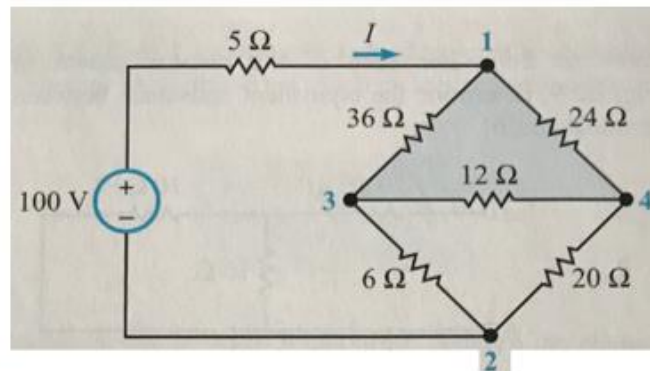
$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$$

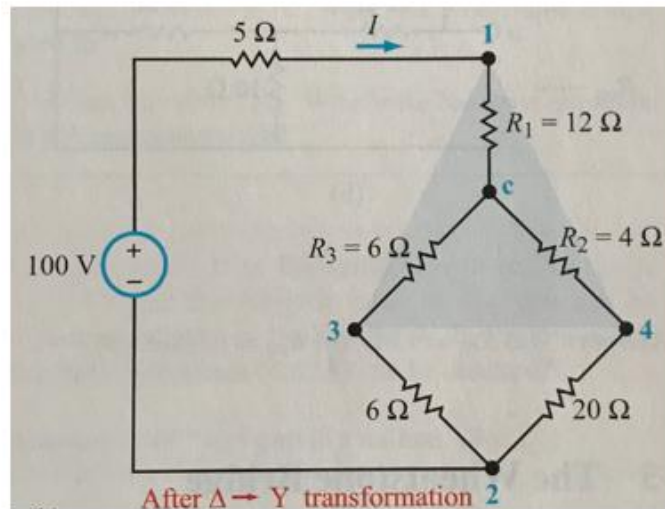
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$

Application of the Y – Δ Transformation

Reduce the following circuit to its most simple form.



First, we replace the Δ circuit connected to nodes 1, 3 and 4 by a Y circuit,



$\Delta \rightarrow Y$ Transformation:

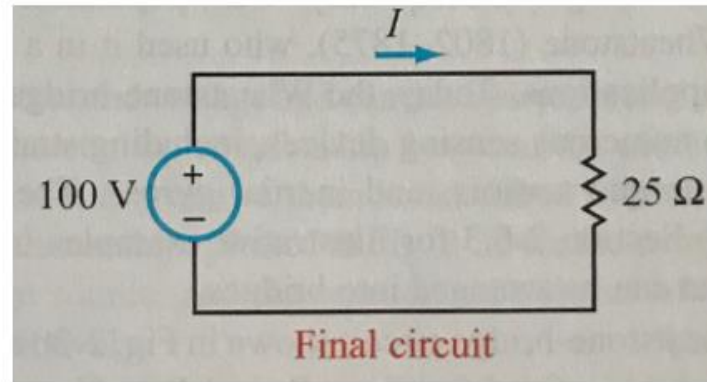
$$R_1 = \frac{(24)(36)}{24 + 36 + 12} = 12\ \Omega$$

$$R_2 = \frac{(24)(12)}{24 + 36 + 12} = 4\ \Omega$$

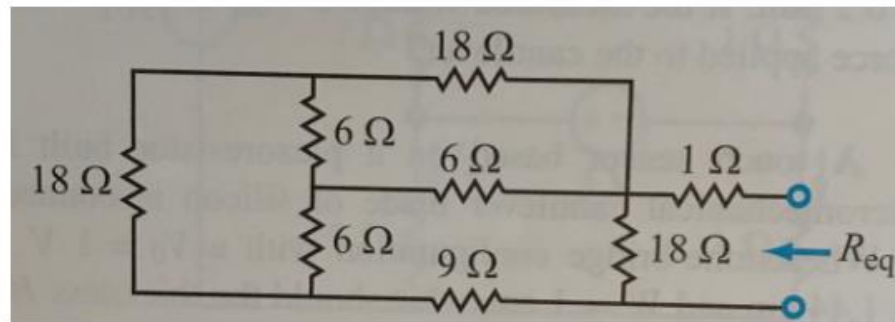
$$R_3 = \frac{(36)(12)}{24 + 36 + 12} = 6\ \Omega$$

Application of the Y – Δ Transformation (Cont.)

The resulting circuit can now be reduced using series and parallel combinations of resistors to obtain (**your turn**: try it):

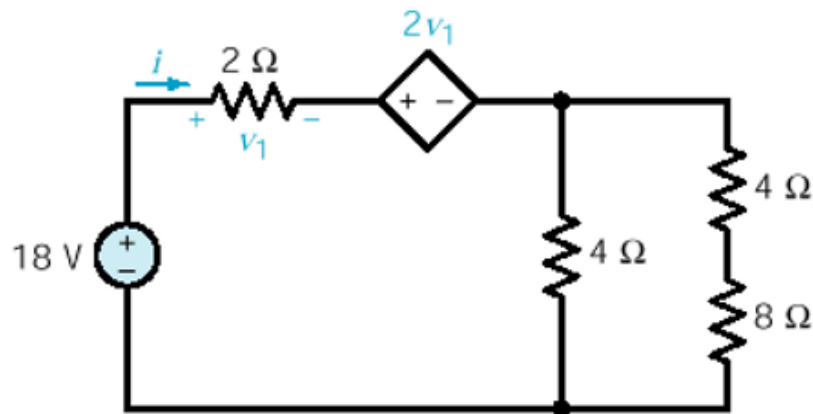


Your turn: Consider the following circuit. Show that $R_{eq} = 10\Omega$.



Example: Resistive Circuit with Dependent Voltage Source

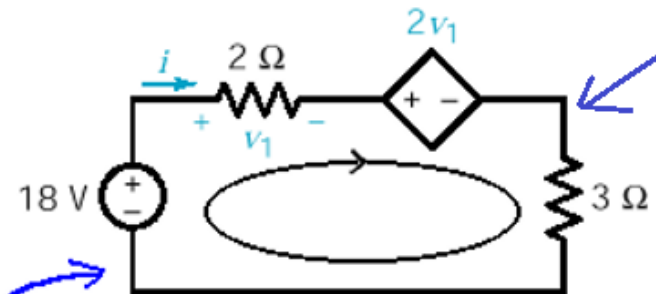
Find the current i .



Upon combining the three resistors, the circuit reduces to:

$$\begin{aligned} 1/R &= 1/4 + 1/(4 + 8) \\ &= 4/12 = 1/3 \end{aligned}$$

The 2 Ohm and 3 Ohm resistors must not be combined. Why?

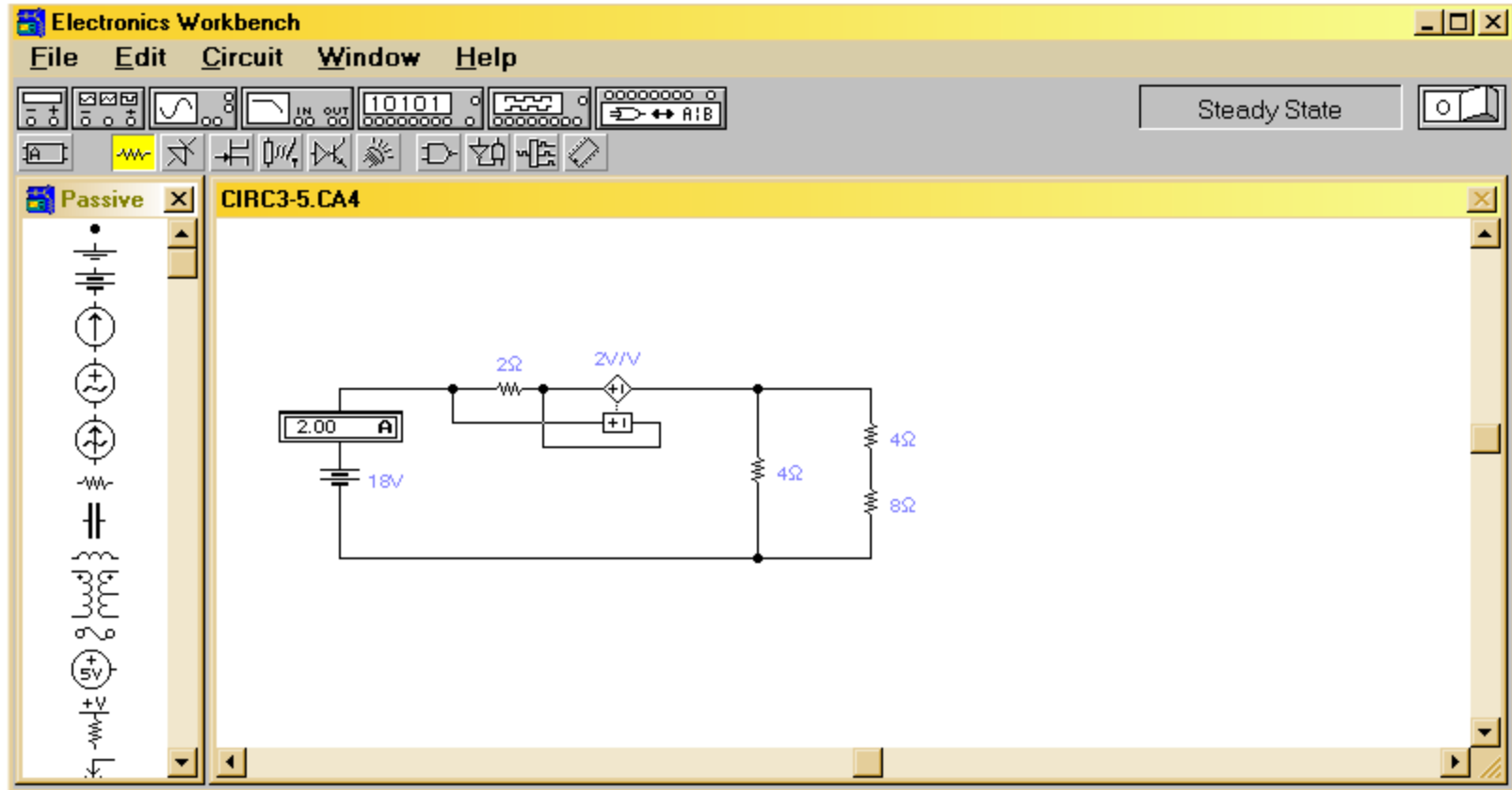


$$\begin{aligned} \text{KVL: } -18 + v_1 + 2v_1 + 3i &= 0 \\ \text{or, by noting that } v_1 &= 2i, \\ -18 + 2i + 2(2i) + 3i &= 0 \end{aligned}$$

$$\text{Therefore, } 9i = 18 \quad \boxed{i = 2 \text{ A}}$$

Example: Resistive Circuit with Dependent Voltage Source

Workbench Solution



Example

Determine R and the power delivered to the 6Ω resistor, when $i = 2\text{A}$.

Solution:

Use **current divider** to obtain R :

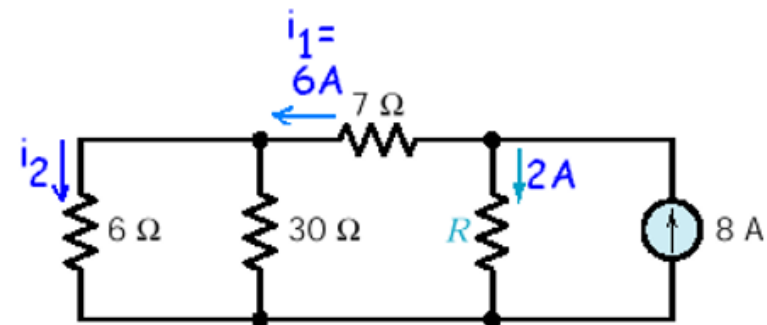
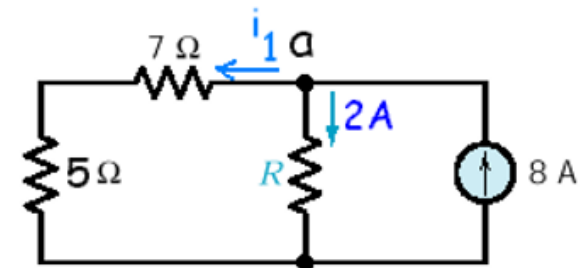
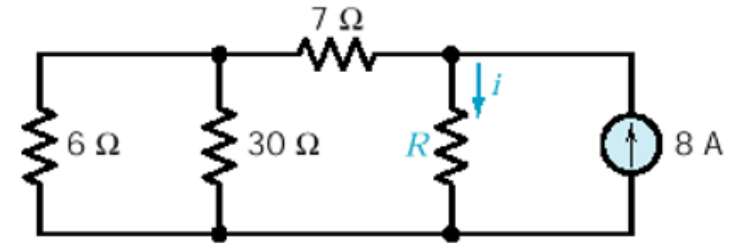
$$2 = 8 \times \frac{7+5}{R+(7+5)} = 8 \times \frac{12}{R+12} \Rightarrow \underline{R = 36\Omega}$$

KCL at a : $i_1 = 8 - 2 = 6\text{A}$

Use **current divider** to obtain i_2 :

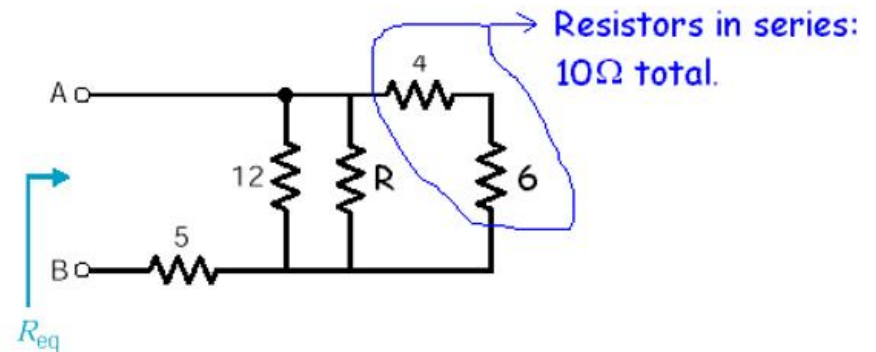
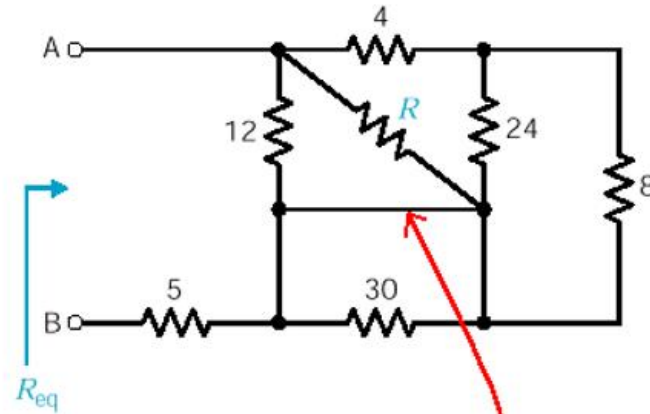
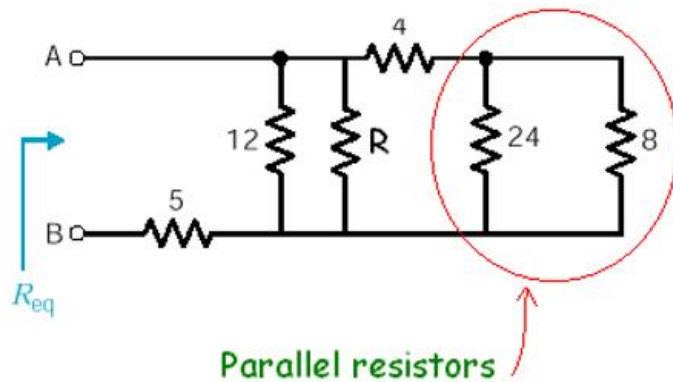
$$i_2 = i_1 \frac{30}{6+30} = 6 \frac{(30)}{36} = 5\text{A}$$

$$\therefore P_{6\Omega} = i_2^2 \cdot 6\Omega = (5\text{A})^2 6\Omega = \underline{150\text{W}}$$



Example

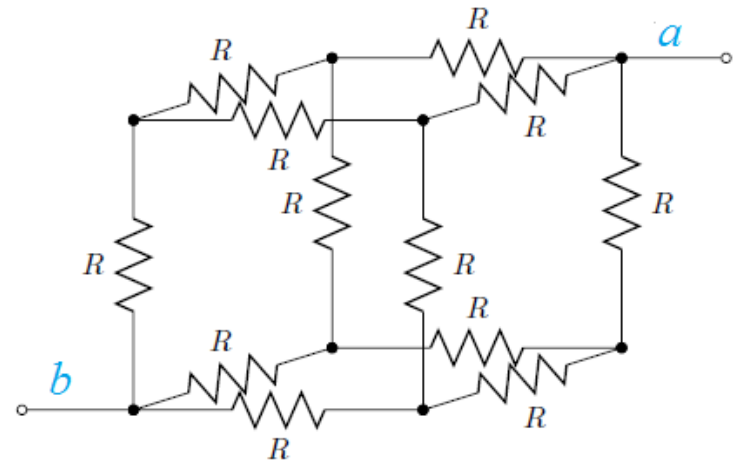
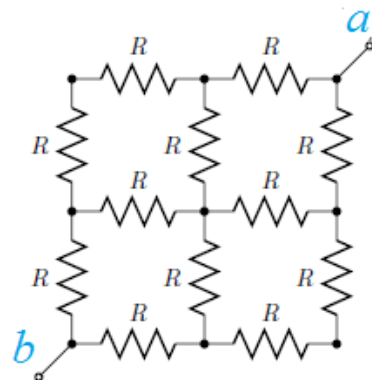
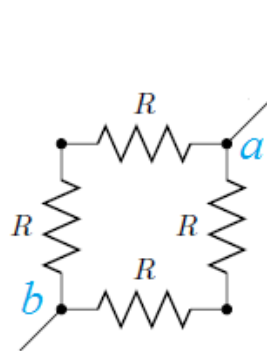
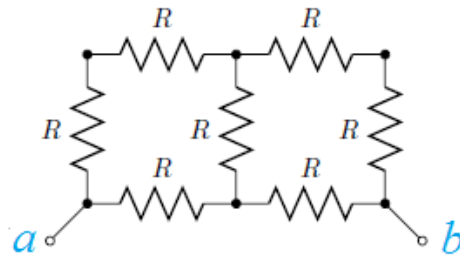
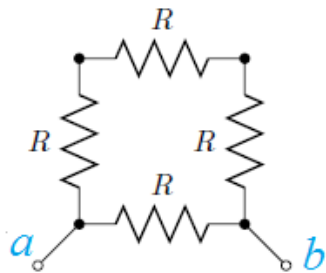
For the following circuit, given that $R_{eq} = 9 \text{ Ohms}$, find R .



$$R_{eq} = 5 + \frac{1}{\frac{1}{12} + \frac{1}{R} + \frac{1}{10}} = 9; \text{ solving for } R \text{ yields } \underline{R = 15\Omega}$$

Problems: Equivalent Resistance of a Network of Resistors

Your turn: Find the equivalent resistance measured between nodes a and b in the following circuits. Hint: Make use of symmetry ideas. You may apply a 1 A current source between a and b and determine the source Voltage drop (employ symmetry reasoning and KVL) then compute R_{eq} using $R_{eq} = V/1$. Also, keep in mind that no current flows in a resistor if the voltages at its two terminals are equal.



[Click here for solution\(s\)](#)

Example

A computer analysis program provides the following branch currents and voltages for the circuit shown: $i_1 = -0.833$, $i_2 = -0.333$, $i_3 = -1.167$, and $v = -2.0$. Are these answers correct?

Solution:

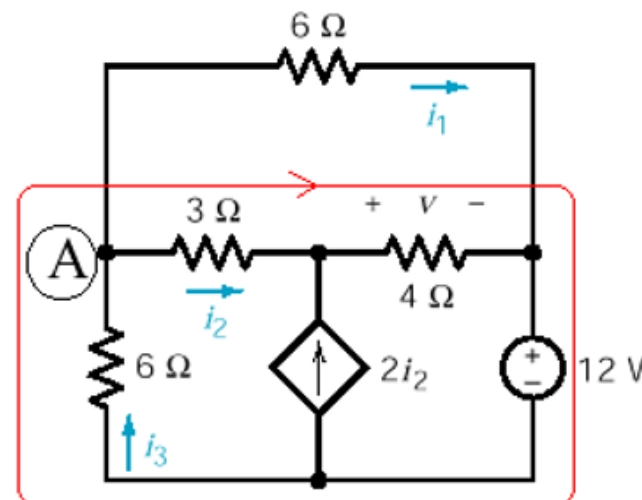
$$\begin{aligned}\text{KCL @ A: } i_3 &= i_1 + i_2 \\ -1.167 &= -0.833 + (-0.333) \\ -1.167 &= -1.166 \quad \boxed{\text{OK}}\end{aligned}$$

KVL around loop shown in red:

$$6i_3 + 3i_2 + v + 12 = 0$$

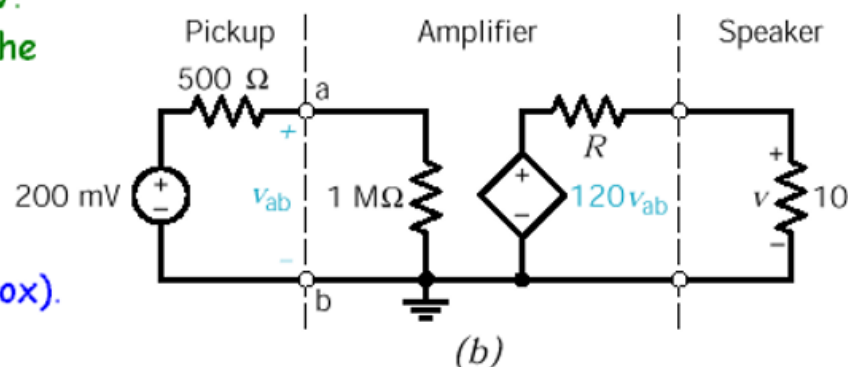
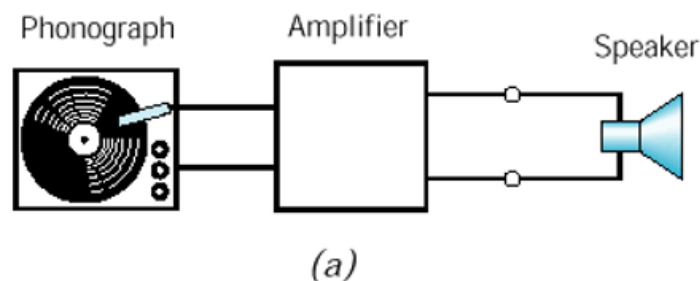
yields $v = -4.0 \text{ V}$ not $v = -2.0 \text{ V}$

The voltage value given is in error.



Example

A phonograph pickup, stereo amplifier, and speaker are shown in (a) and redrawn as a circuit model in (b). Determine the resistance R so that the voltage v across the load is 16 V. Determine the power delivered to the speaker.



By voltage division:

$$V_{ab} = \frac{10^6}{10^6 + 500} 0.2 = 0.2 \text{ V (approx).}$$

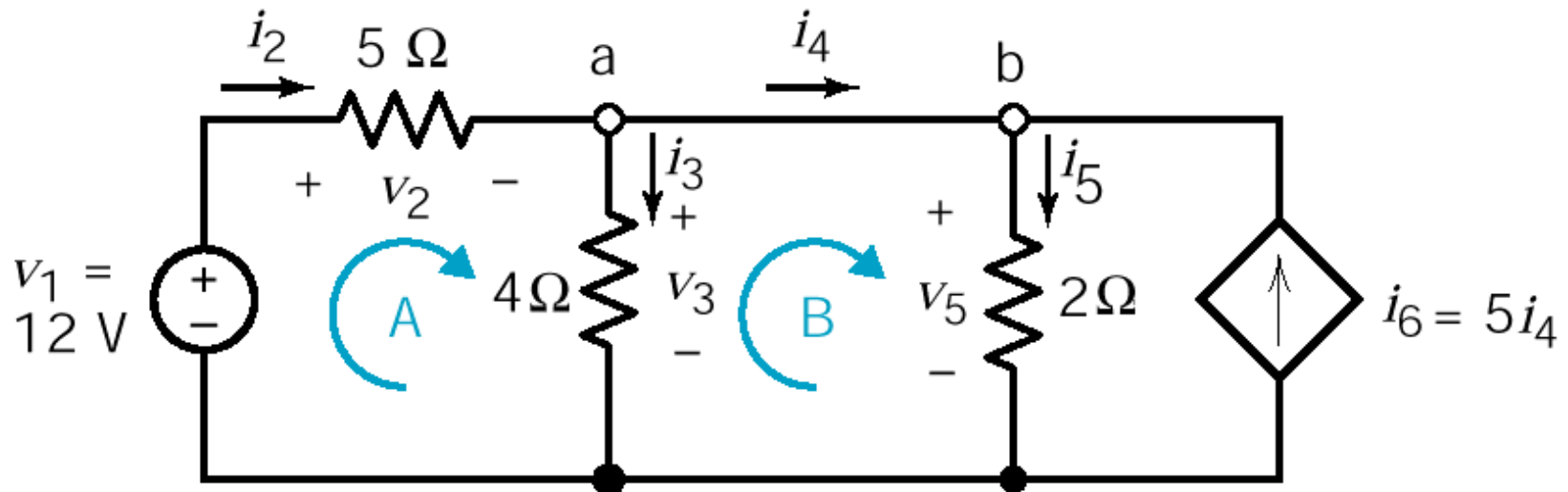
Similarly, by voltage division at RHS loop

$$v = \frac{10}{10 + R} 120v_{ab} = \frac{10}{10 + R} (120) (.2)$$

$$\text{let } v = 16 = \frac{240}{10 + R} \Rightarrow \underline{R = 5\Omega} \quad \therefore P = \frac{16^2}{10} = \underline{25.6W}$$

A More General Example (more on this in the next chapter)

Find v_2 , i_3 , i_4 , and v_5 .



i_2 , i_5 , and v_3 can be eliminated by using Ohm's law:

$$\text{KVL (A): } -12 + v_2 + 4i_3 = 0$$

$$\text{KVL (B): } -v_3 + v_5 = 0$$

$$\text{KCL (a): } i_2 - i_3 - i_4 = 0$$

$$\text{KCL (b): } i_4 - i_5 + 5i_4 = 0$$

$$i_2 = \frac{v_2}{5}, \quad i_5 = \frac{v_5}{2}, \quad \text{and } v_3 = 4i_3$$

$$v_2 + 4i_3 = 12$$

$$v_5 = 4i_3$$

$$\frac{v_2}{5} = i_4 + i_3$$

$$\frac{v_5}{2} = 6i_4$$

Solving by substitution we get:

$$v_2 = 7.5 \text{ V} \quad i_3 = 1.125 \text{ A}$$

$$i_4 = 0.375 \text{ A} \quad v_5 = 4.5 \text{ V}$$

Electronics Workbench

Mathcad

Example (cont). Mathcad Solution

given

$$v_2 + 4i_3 = 12$$

$$v_5 = 4i_3$$

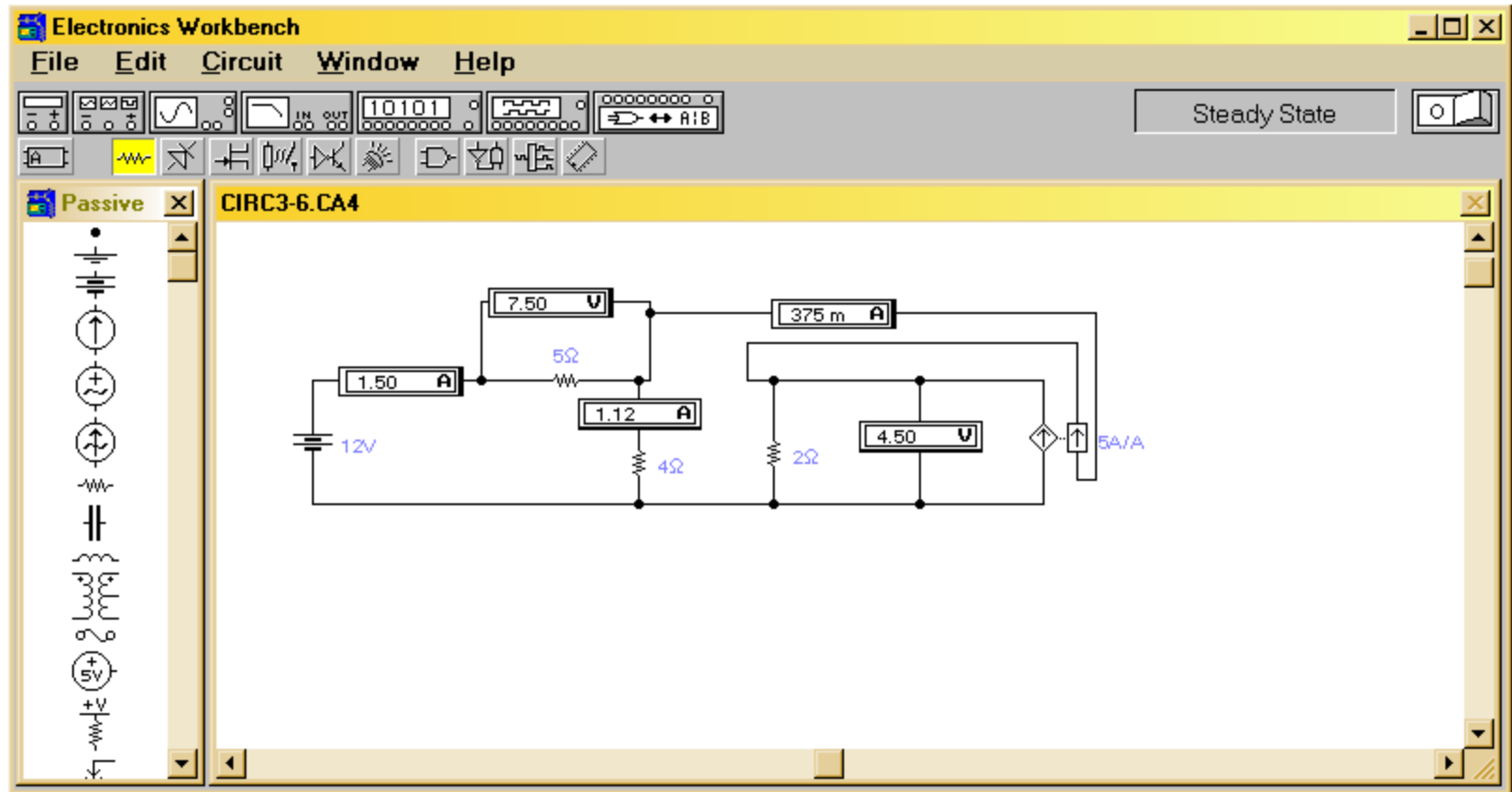
$$\frac{v_2}{5} = i_4 + i_3$$

$$\frac{v_5}{2} = 6i_4$$

find(v_2, i_3, i_4, v_5) $\rightarrow$

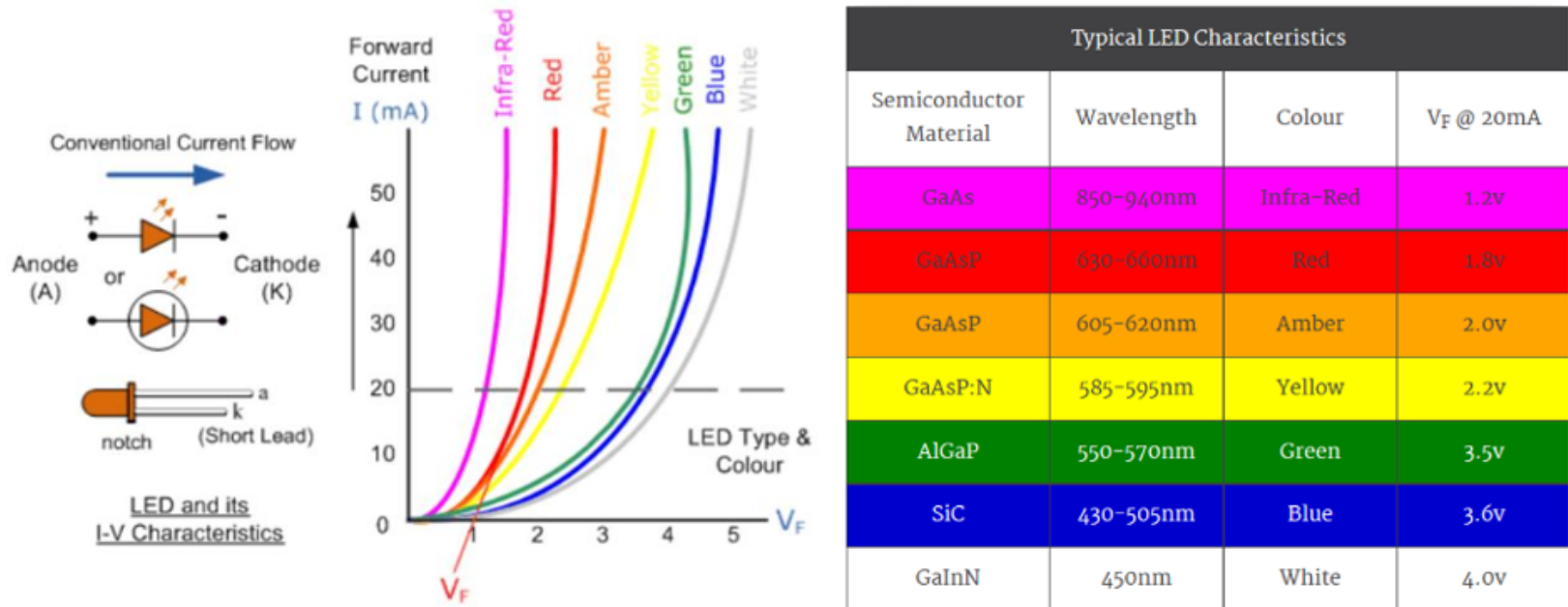
$$\begin{pmatrix} \frac{15}{2} \\ 9 \\ 8 \\ 3 \\ 8 \\ 9 \\ 2 \end{pmatrix}$$

Example (cont). Workbench Solution



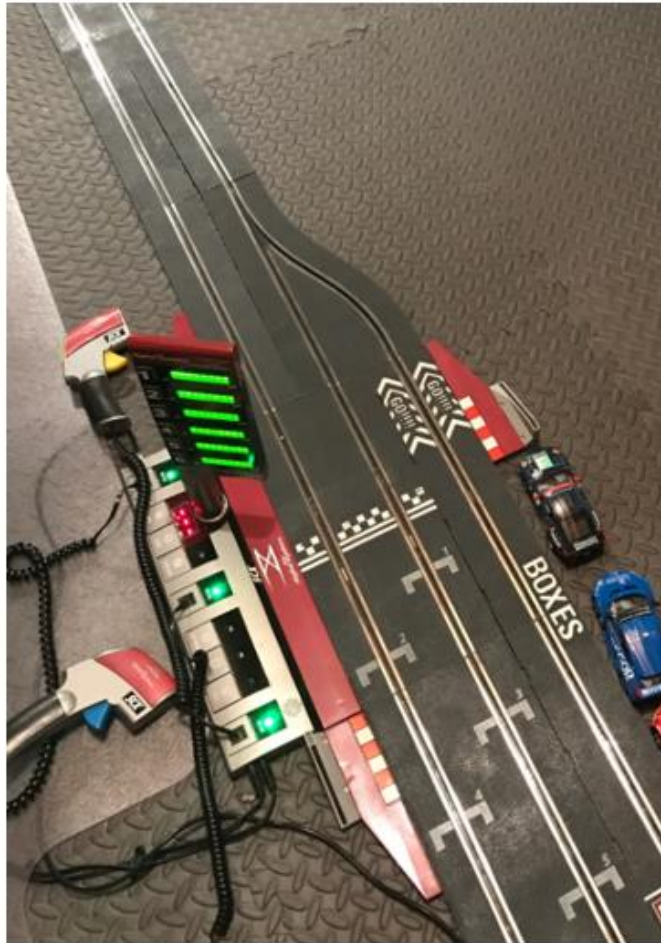
Slot Car LED Headlights Circuit Design Problem

Consider the problem of designing a circuit to control the headlights of a toy slot car. We would like to use two white light emitting diodes (LEDs) as the headlights. The LED blocks a current trying to flow through it in the reverse direction. From the following chart we see that the white LED produces bright light if its forward current is about 20mA. The forward voltage drop in this case is 4 Volt.



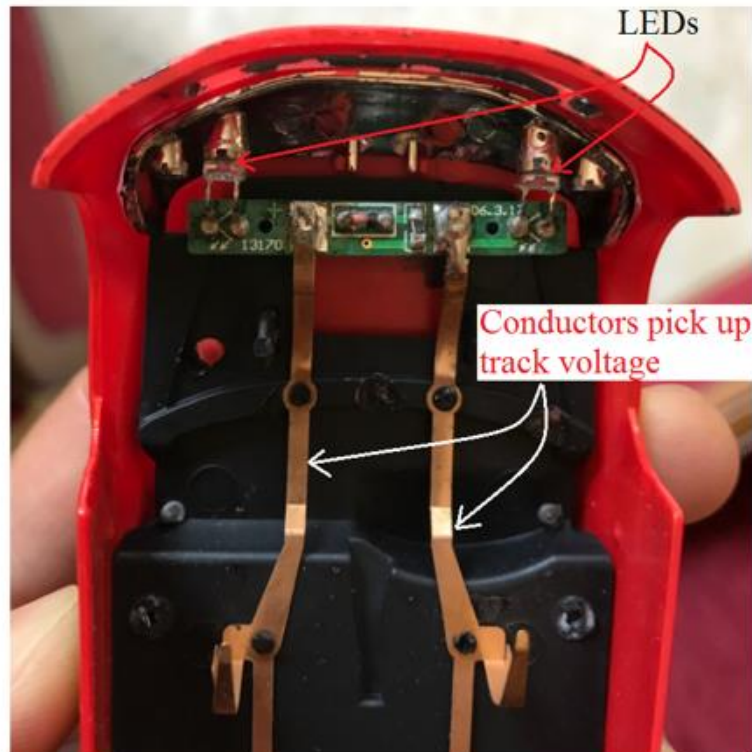
Slot Car LED Headlights Circuit Design Problem

The following figures show a slot car track and a typical slot car.



Slot Car LED Headlights Circuit Design Problem

Using a voltmeter we measure a track voltage of about 18 Volt. This is the voltage source that will power the two LEDs.

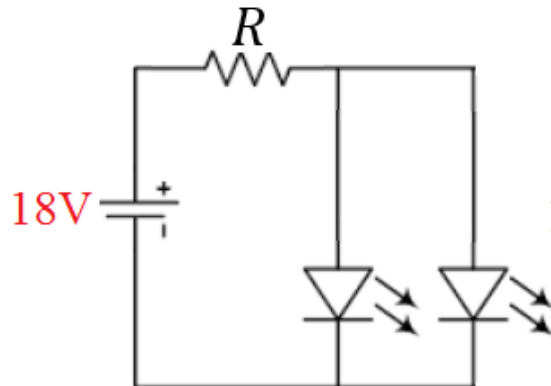


Track Voltage

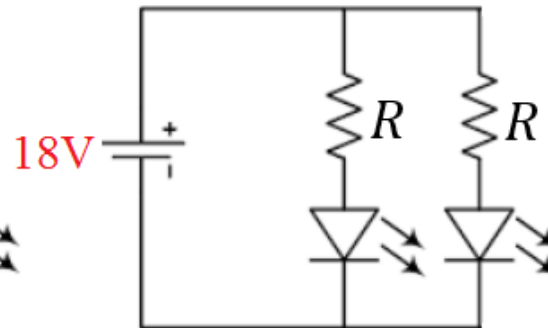


Slot Car LED Headlights Circuit Design Problem

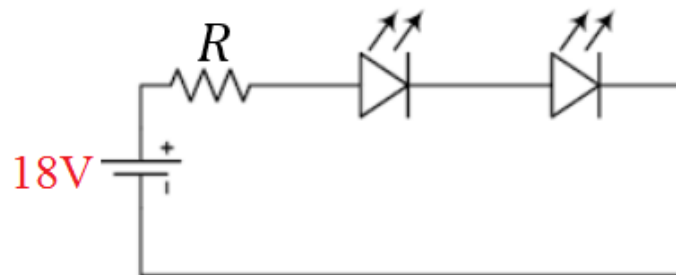
There are different circuits we can use to realize our headlights circuit, as shown below. We will use the circuit in figure (a).



(a)



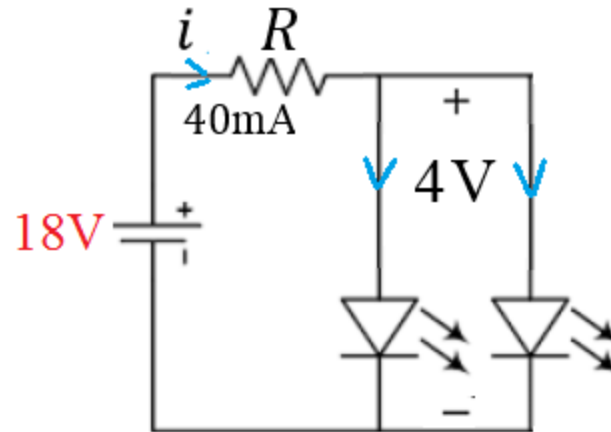
(b)



(c)

Slot Car LED Headlights Circuit Design Problem

The following circuit displays all the known currents and voltages.



Writing the KVL equation around the loop gives:

$$-18 + 0.04R + 4 = 0$$

Standard 5% resistor
values

Therefore the resistor required is $R = 350\Omega$. However, the closest discrete resistor available is a 330Ω or 360Ω . We will select the 330Ω resistor (close enough).

Design Example Continued: Power Rating of Resistor

We must select the 330Ω resistor so that it can handle the dissipated energy. The power dissipated in the resistor is (by Joule's Law) $P = RI^2 = (330)(0.04)^2 = 0.528\text{W}$. Knowing that resistors are available in (max) power rating of $1/8\text{W}$, $1/4\text{W}$, $1/2\text{W}$ and 1W , we select our resistor to have a power rating of 1W .

Slot Car LED Headlights Circuit Design Problem

Your turn:

Can you tell which circuit (a-c in the above figure) requires a lower power rated resistor? Why?

Repeat the design problem for circuit (b) and then for circuit (a). Discuss the advantages and disadvantages of each circuit.

Case Study: The Transatlantic Cable Fault

The following is an elegant application of a simple resistive circuit employing the *Blavier method* (Edouard E. Blavier, 1826-1887). In 1871, the English mathematical electro-physicist Oliver Heaviside (1850-1925) had recorded in one of his notebooks an application of *Blavier method*. Heaviside was working as a telegraph operator in Denmark when he was faced with the problem of locating a leakage fault that occurred somewhere along the 360 mile undersea cable between Sondervig in Denmark (end A) and Newbiggin in England (end B). The cable had a resistance of 6 Ohms per mile. Most likely, a formidable deep sea creature had bit through the telegraph cable iron sheathing and had exposed the conductive wire, thus introducing a resistance (due to sea water) between the cable and ground.

Case Study: The Transatlantic Cable Fault (cont.)

Heaviside performed two resistance measurements of the cable at end A: One with end B open that resulted in $R_o = 1,040$ Ohms and another with end B shorted (to ground) that resulted in $R_s = 970$ Ohms. Let the intact cable resistance be $R = (360)(6) = 2,160$ Ohms. In order to determine the resistance R_f between end A and the fault, Heaviside used Blavier's formula,

$$R_f = R_s - \sqrt{(R - R_s)(R_o - R_s)}$$

and determined that the $R_f = 681.38$ Ohms from which he concluded that the fault is at $681.38/6 = 113.56$ miles away from end A at Sondervig.

Your turn: Draw a circuit diagram for the cable problem and derive Blavier's formula. Also, provide a formula for the leakage resistance in terms of R , R_s and R_o .



Are Christmas light connected in parallel or in series?

If you remove one light bulb then all the other lights go off? This means that the lights are in series. So, each light has an effective voltage drop of $120/50=2.4\text{V}$ in a 50 light bulb string.

The old fashioned light strings used incandescent light bulbs each dissipating about 0.425 W. The newer LED strings utilize more energy efficient colored LEDs. Depending on the color, the LED light is supposed to drop 1.8 to 3 volts. A red LED at 1.8 Volt has about 20mA current leading to a dissipation of 0.036 W

If the light bulbs are in series, then how come if one or few of those lights blowup (but remain plugged into the string) the remaining lights can still work? Would the remaining lights look brighter? The answer to such questions can be found here:

<https://www.wired.com/2014/12/christmas-lights-series-parallel/>
<http://main.flosscience.com/Home/clever-hacks/xmas-lights>



What is the resistance of a 100W incandescent light bulb used in a USA household?

Well, the light bulb operates at an effective (dc equivalent for average power calculations in resistors) voltage of 120V. Therefore the resistance of the 100W light bulb is

$$R = \frac{120^2}{100} = 144\Omega$$

But, how come if you are to use an Ohmmeter, you would only measure (at room temperature of $T_o = 20\text{ C}^\circ$) about $R_o = 12\Omega$ resistance for the 100W light bulb?

Hint: The filament of the incandescent light bulb is made out of Tungsten with temperature coefficient $\alpha = (4.5)10^{-3}\text{ (C}^\circ)^{-1}$. The operating temperature of a 100W light bulb is around 2400 C° . Resistance varies with temperature as,

$$R = R_o[1 + \alpha(T - T_o)]$$