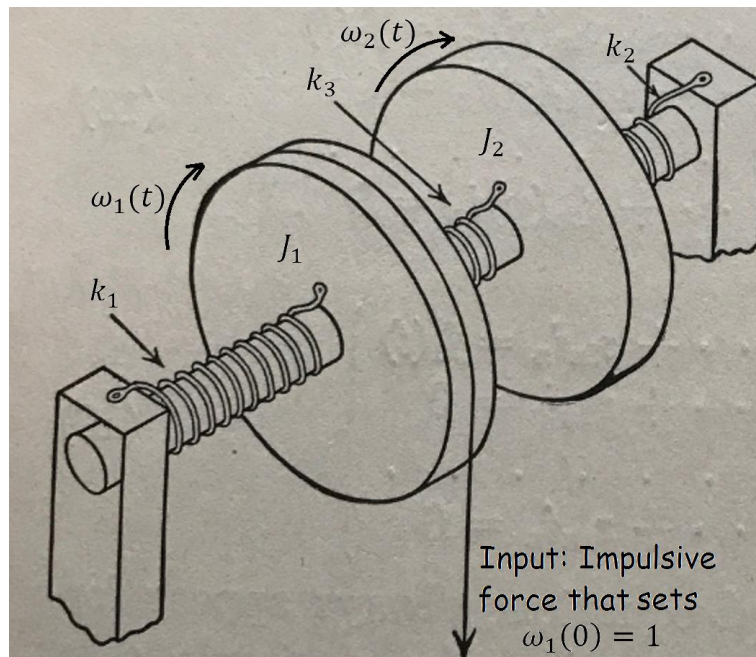


One Last "Your Turn" Problem

Consider the frictionless mechanical system shown in the figure below. The flywheels are loose on the shaft; and each is attached to a helical spring, the other end of which is fixed. A third spring connects the two flywheels.

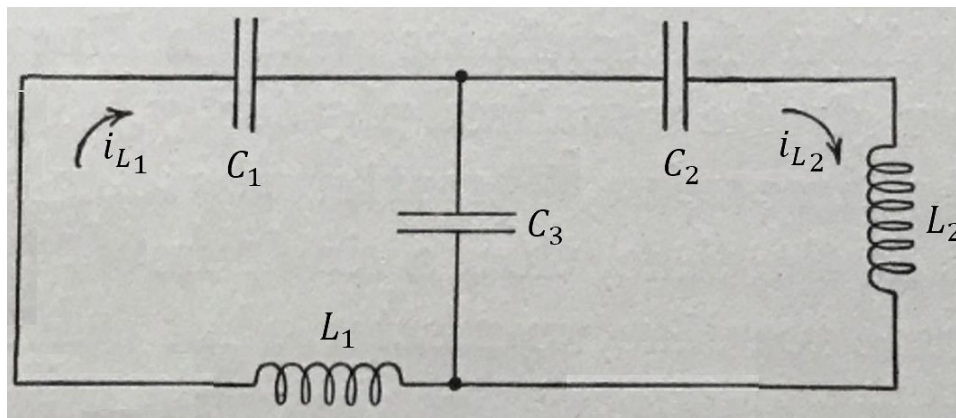


Let J_1 and J_2 represent the rotational moments of inertial of the two flywheels. Let k_1 , k_2 and k_3 represent the torsion spring constants of the three springs. Let the angular velocities of the flywheels be $\omega_1(t)$ and $\omega_2(t)$, with corresponding angular positions of $\theta_1(t) = \int_0^t \omega_1(\tau) d\tau$ and $\theta_2(t) = \int_0^t \omega_2(\tau) d\tau$, respectively. [We assume $\theta_1(0) = \theta_2(0) = 0$.]

The system starts at rest at $t < 0$ (zero initial stored energy). At $t = 0$ the first flywheel receives an impulsive force that sets its initial angular velocity to $\omega_1(0) = 1$ rad/sec. You are asked to solve for and plot $\omega_1(t)$ and $\theta_1(t)$ with respect to time. Also, you are asked to generate a phase plane plot of $\omega_1(t)$ vs $\theta_1(t)$.

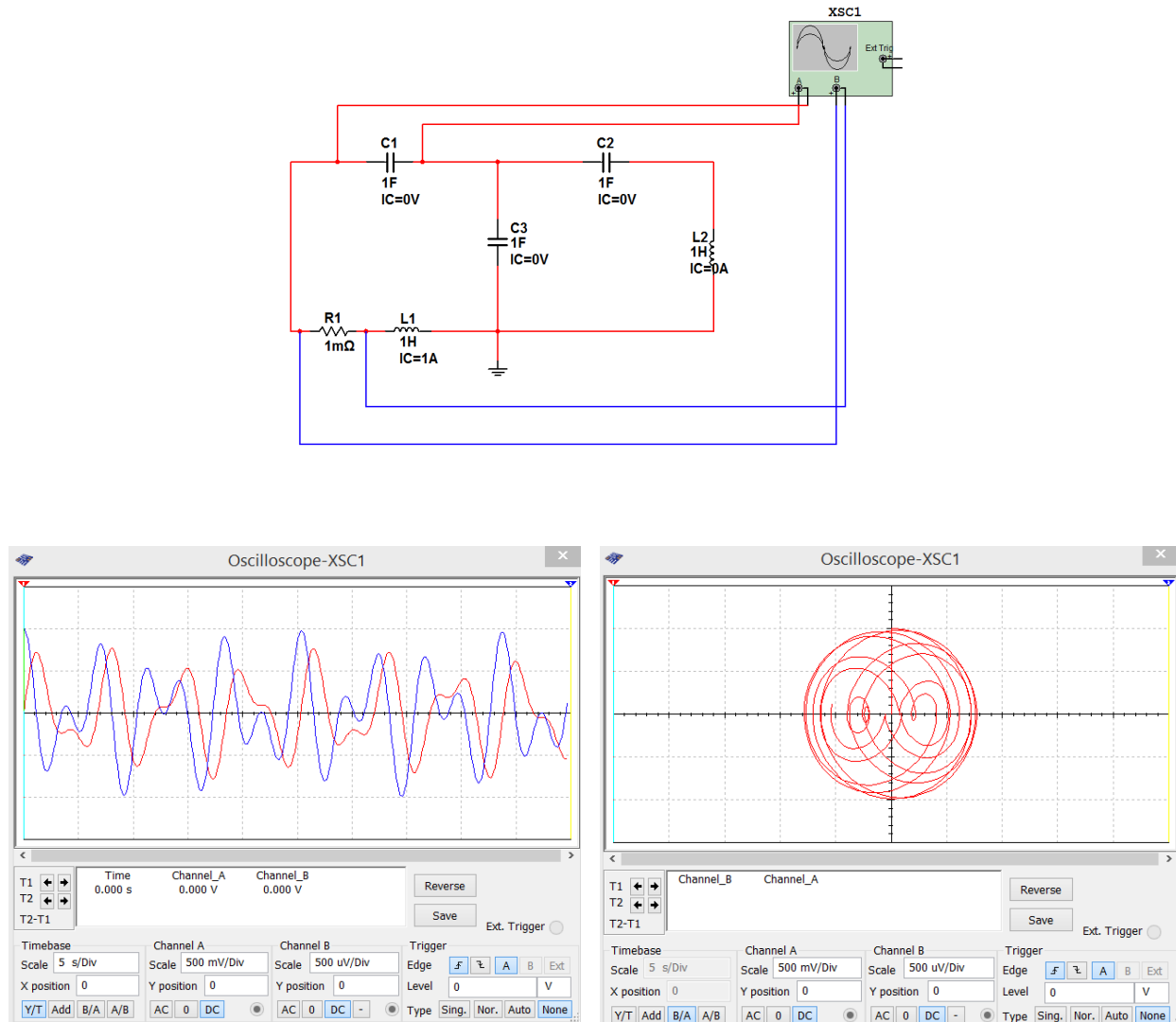
A friend of yours (who happened to be a Junior student pursuing a physics/EE double major) suggested that the following circuit (refer to the figure below) is an analog to the above mechanical system.

Alternatively, you don't have such a friend but you have a copy of the classic book "Operational Circuit Analysis" by Vannevar Bush (Wiley, 1929) and you read Pages 24-25. The circuit is a 4th order circuit (it has five energy storing element, but the coupling capacitor does not increase the order of this particular circuit; so, effectively it has four independent energy storing elements making it 4th order. You should expect this circuit then to have four natural frequencies. And because it has no dissipative elements (resistors) then its responses are oscillatory in nature.)

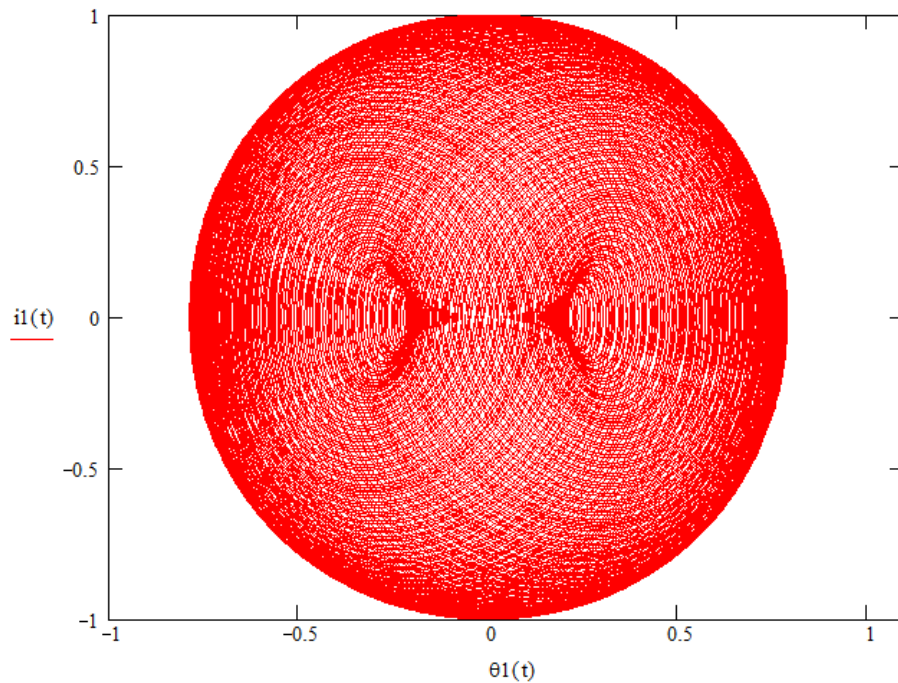


You were told that the following quantities of the mechanical and electrical models correspond to each other: $L_1 \equiv J_1$, $L_2 \equiv J_2$, $C_1 \equiv k_1$, $C_2 \equiv k_2$, $C_3 \equiv k_3$, $i_{L_1}(t) \equiv \omega_1(t)$ and $i_{L_2}(t) \equiv \omega_2(t)$. You then observed that the first capacitor voltage must be equivalent to the angular position $\theta_1(t)$ of the first flywheel because you can write $v_{C_1}(t) = \frac{1}{C_1} \int_0^t i_{L_1}(\tau) d\tau$. So, for you, the problem becomes the determination of $i_{L_1}(t)$ and $v_{C_1}(t)$ with the initial condition $i_{L_1}(0^-) = 1$ Amp. You may set the circuit element values to $L_1 = L_2 = 1\text{H}$ and $C_1 = C_2 = C_3 = 1\text{F}$ in your analysis.

Before solving the problem analytically in the s -domain you take advantage of Multisim and (**your turn:**) generate the required plots on a virtual oscilloscope to obtain the following results.



Notes: A very small resistor is placed in series with L_1 so we may use its voltage (proportional to its current) to indirectly display i_{L_1} on the scope. The scope X-Y plot shown in the figure to the right is a plot of $i_{L_1}(t)$ versus $v_{C_1}(t)$. The plot is bounded but never repeats. It is referred to as a space filling curve. If we let the trace continue for large t we will see the following scope display.



Your turn: Now you are left with applying KVL to write the s -domain equations for the circuit (and yes, you can take advantage of Mathcad to solve the equations, transform the solutions to the time domain and generate your plots). What is the order of the circuit and what are the natural frequencies? *I hope you have enjoyed ECE3300!*