

ECE 3300
Electrical Circuits: Laboratory
Experiment 4 Pre-Reading
Simple Resistive Circuits
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This experiment deals with current and voltage measurements in simple resistive circuits. First, the computation and measurement of the equivalent resistance of a network of resistors is presented. Next, theoretical expressions are derived for the voltage divider and the current divider circuits; these expressions will be verified in the lab. The potentiometer is introduced as a voltage divider with a variable division ratio. Finally, a simple thermistor circuit is described and is used for sensing temperature.

Learning Objectives

By the end of this unit, the student should be able to build and test simple resistive circuits. More specifically, the student should be able to:

- Use an Ohmmeter to measure the equivalent resistance of a network of resistors.
- Build a current divider circuit.
- Measure resistor currents in a current divider circuit.
- Build a voltage divider circuit.
- Measure the voltage drop across resistors in a voltage divider circuit.
- Use a potentiometer to build a variable-division-ratio voltage divider.
- Explain the effect of large resistor values on the accuracy of voltage measurements.
- Explain the effects of small resistor values on the accuracy of current measurements.
- Build a temperature sensor utilizing a thermistor.
- Derive the mathematical expression that relates temperature to the voltage drop across a thermistor.

Equivalence of a Resistive Network

Circuits with multiple resistors are common in many applications. Resistors can appear connected in *series*, in *parallel* and in other configurations. The

analysis of a given resistive circuit can be simplified if two or more of its resistors are substituted by a single *equivalent resistor*.

Figure 1 shows two resistor, R_1 and R_2 , connected in series. A single resistor R is said to be *equivalent* to the two *series* resistors, if $R = R_1 + R_2$. Note that if $R_1 \gg R_2$, then $R \cong R_1$.

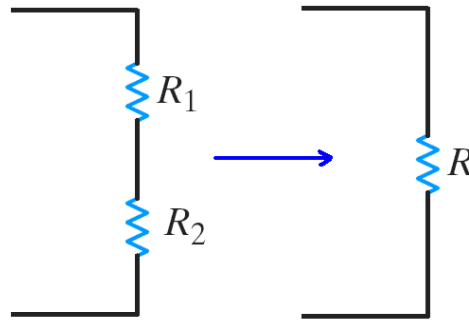


Figure 1. Two series resistors can be combined as a single resistor.

The equivalent resistance for N resistors, $R_1, R_2, \dots R_N$, connected in series is:

$$R_{eq} = R_1 + R_2 + \dots + R_N.$$

Figure 2 shows two resistor, R_1 and R_2 , connected in parallel. A single resistor R is said to be *equivalent* to the two *parallel* resistors, if $1/R = 1/R_1 + 1/R_2$ or $R = R_1 R_2 / (R_1 + R_2)$. Note that if $R_1 \gg R_2$, then $R \cong R_2$.

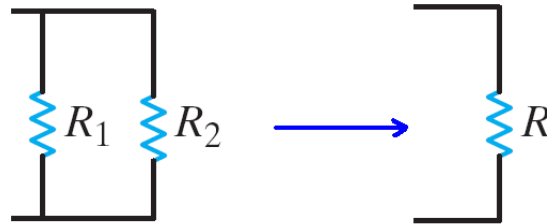


Figure 2. Two parallel resistors can be combined as a single resistor.

The equivalent resistance for N resistors, $R_1, R_2, \dots R_N$, connected in parallel is:

$$1/R_{eq} = 1/R_1 + 1/R_2 + \dots + 1/R_N.$$

Figure 3-a shows a network of three resistors. The equivalent resistance between nodes a and b can be determined as follows: First, combine the two

1k Ω parallel resistors into a single 0.5k Ω resistor. This results in a network consisting of a 1k Ω resistor in series with a 0.5k Ω resistor (refer to Figure 3-b). The resulting network can be further reduced to a single resistor by adding the resistances to obtain $R_{eq} = 1.5\text{k}\Omega$ (as shown in Figure 3-c).

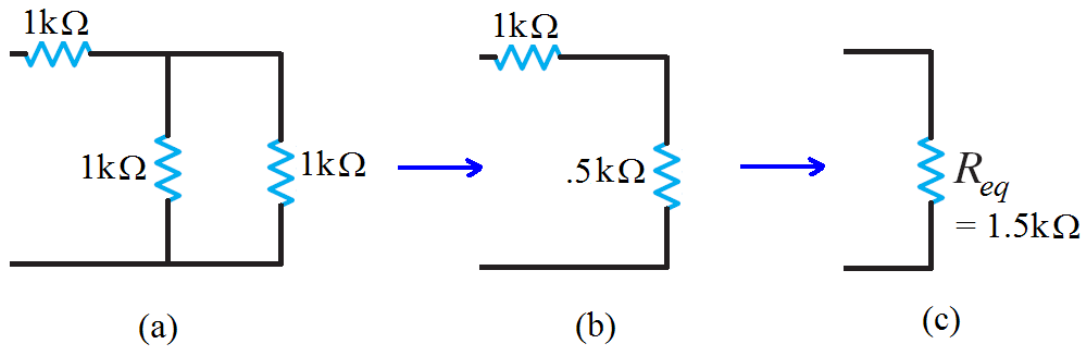


Figure 3. Reducing a resistive network to a single equivalent resistor.

Figure 4 shows a network of six resistors. None of the individual resistors are connected in series or in parallel. Here, one can easily determine the equivalent resistance of the network using an Ohmmeter. Assuming 1k Ω resistors, Multisim's virtual Ohmmeter can be used to determine the network's equivalent resistance between nodes *a* and *b*; the answer (2k Ω) is shown in Figure 5.

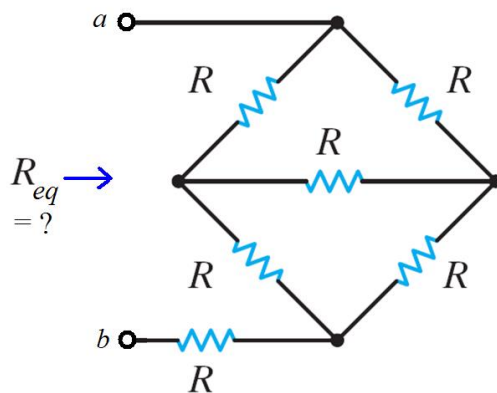


Figure 4. A resistive network.

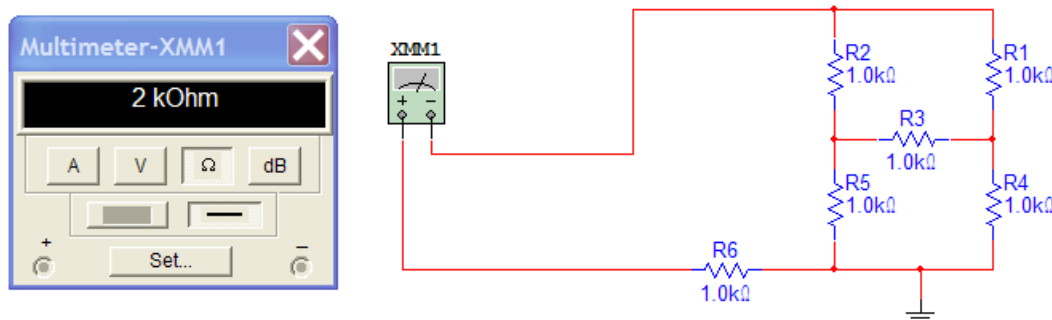


Figure 5. Multisim's solution for the equivalent resistance of the network in Figure 4.

Current Divider Circuit

A current divider circuit is shown in Figure 6. It consists of a current source, $i(t)$, connected in parallel with two resistors, R_1 and R_2 . The current $i(t)$ is divided into the currents $i_1(t)$ and $i_2(t)$ that flow in R_1 and R_2 , respectively. The voltage drop across both resistors, by Ohms law, is $v(t) = R_1 i_1(t) = R_2 i_2(t)$. (This implies that the resistor with the higher resistance has the smaller current.)

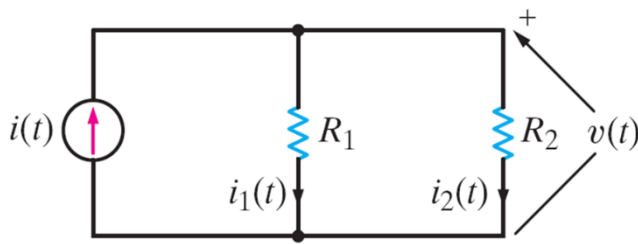


Figure 6. Current divider circuit.

Now, replace the resistors by their equivalent single resistor R , $R = R_1 R_2 / (R_1 + R_2)$, then, the current through and the voltage across R would be $i(t)$ and $v(t)$, respectively, where $v(t) = R i(t) = R_1 R_2 i(t) / (R_1 + R_2)$. Combining the above results, allows us to write:

$$R_1 i_1(t) = R_1 R_2 i(t) / (R_1 + R_2) \text{ and } R_2 i_2(t) = R_1 R_2 i(t) / (R_1 + R_2),$$

from which we can solve for the individual currents (*current divider currents*) to obtain:

$$i_1(t) = \frac{R_2 i(t)}{R_1 + R_2} \text{ and } i_2(t) = \frac{R_1 i(t)}{R_1 + R_2} \quad (1)$$

We may verify these equations by trying extreme values for one of the resistors; say, R_1 approach zero or R_1 approach infinity. If R_1 is zero (a piece of wire), no current will flow through R_2 ; all electrons in the current $i(t)$ will choose the zero resistance path. This means that $i_1(t) = i(t)$. On the other hand, if R_1 is very large (infinite), it will behave as an open circuit and zero current will flow through it. This leads to $i_2(t) = i(t)$.

Figure 7 depicts the circuit model for a current divider with an ammeter. The ammeter is inserted in the circuit to measure the current $i_1(t)$. Recall that the ammeter has a small (few Ohms) input resistance, R_A . The ammeter resistance is in series with R_1 . This results in a current divider with two parallel resistors: $(R_A + R_1)$ and R_2 . According to the expression in (1), the current values are now given by:

$$i_1(t) = \frac{R_2 i(t)}{R_1 + R_A + R_2} \text{ and } i_2(t) = \frac{(R_1 + R_A) i(t)}{R_1 + R_A + R_2} \quad (2)$$

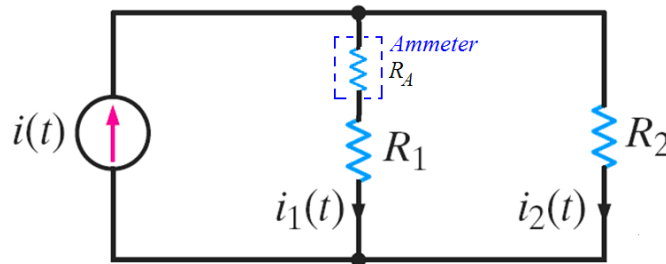


Figure 7. Model of an ammeter in a current divider circuit.

Therefore, the placement of the ammeter will alter the circuit and may lead to inaccurate current measurements. The errors are significant if R_1 is small (say, $R_1 < 50\Omega$). On the other hand, the effect of the ammeter can be ignored if $R_1 \gg R_A$.

Voltage Divider Circuit

A *voltage divider* circuit is shown in Figure 8. It consists of a voltage source, $v(t)$, connected in series with two series resistors, R_1 and R_2 . The voltage $v(t)$ is divided into the two voltages $v_1(t)$ and $v_2(t)$ across R_1 and R_2 , respectively.

The current through both resistors, by Ohms law, is $i(t) = v_1(t)/R_1 = v_2(t)/R_2$. This means that the resistor with the higher resistance has a higher voltage drop across its terminals.

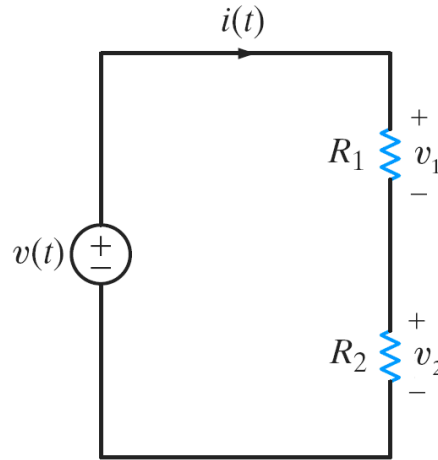


Figure 8. Voltage divider circuit.

Now, replace the resistors by their single equivalent resistor R , $R = R_1 + R_2$. Then, the current through and the voltage across R would be $i(t)$ and $v(t)$, respectively, where $i(t) = v(t)/R = v(t)/(R_1 + R_2)$. Combining the above results, allows us to write:

$$v_1(t)/R_1 = v(t)/(R_1 + R_2) \text{ and } v_2(t)/R_2 = v(t)/(R_1 + R_2),$$

from which we can solve for the individual voltage drops (*voltage divider voltages*) to obtain:

$$v_1(t) = \frac{R_1 v(t)}{R_1 + R_2} \text{ and } v_2(t) = \frac{R_2 v(t)}{R_1 + R_2} \quad (3)$$

We may verify these equations by trying extreme values for one of the resistors; say, R_1 approach zero or R_1 approach infinity. If R_1 is zero (a piece of wire), then the full source voltage will appear across R_2 , or $v_2(t) = v(t)$. On the other hand, if R_1 is very large (infinite), it will behave as an open circuit and zero current will flow through the circuit. This leads to $v_2(t) = R_2 i(t) = 0$.

Figure 9 depicts the circuit model for a voltage divider with a voltmeter. The voltmeter is connected across R_1 to measure voltage $v_1(t)$. Recall that the

voltmeter has a large (approximately $10\text{M}\Omega$) input resistance, R_V . The voltmeter resistance is in parallel with R_1 . This results in a voltage divider with two series resistors: $R_V R_1 / (R_V + R_1)$ and R_2 . According to the expressions in (3), the voltage drops across R_1 and R_2 can be shown to be:

$$v_1(t) = \frac{R_V R_1 v(t)}{R_V R_2 + R_1 (R_2 + R_V)} \quad \text{and} \quad v_2(t) = \frac{R_2 (R_1 + R_V) v(t)}{R_V R_2 + R_1 (R_2 + R_V)} \quad (4)$$

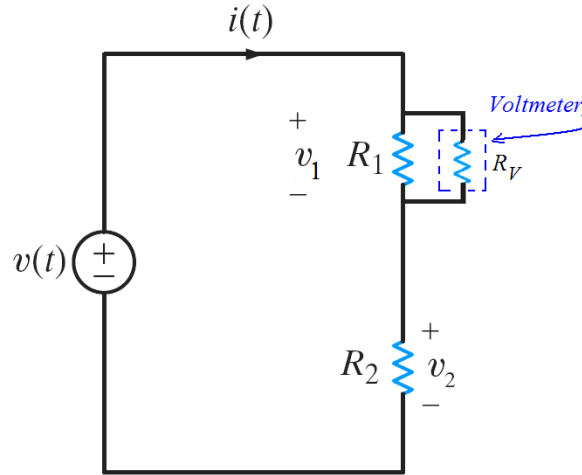


Figure 9. Model of a voltmeter in a voltage divider circuit.

Therefore, the placement of the voltmeter will alter the circuit and may lead to inaccurate voltage measurements. The errors are significant if R_1 is large (say, $R_1 > 1\text{M}\Omega$). On the other hand, the effect of the voltmeter can be ignored if $R_1 \ll R_V$.

The Potentiometer

A *potentiometer* is a three terminal variable resistance circuit component. The terminals of a potentiometer are labeled a , b and c in Figure 10. The total resistance R is a (fixed) resistance measured between terminal a and terminal c . The resistance R_1 between terminal a and b is variable, and can be set by rotating a screw or a knob on the potentiometer. R_1 can be set to a value from 0 to R . The resistance between the b and c terminals decreases as R_1 increases, and is given by $R - R_1$.

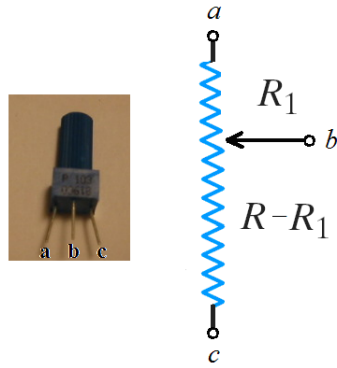


Figure 10. Potentiometer.

Figure 11 depicts a voltage divider based on a potentiometer. The voltage across each resistor is determined by the voltage division formula given in Equation (3). For example, the voltage labeled v_1 in Figure 11 is determined according to

$$v_1(t) = \frac{R_1 v(t)}{R_1 + (R - R_1)} = \frac{R_1}{R} v(t) \quad (5)$$

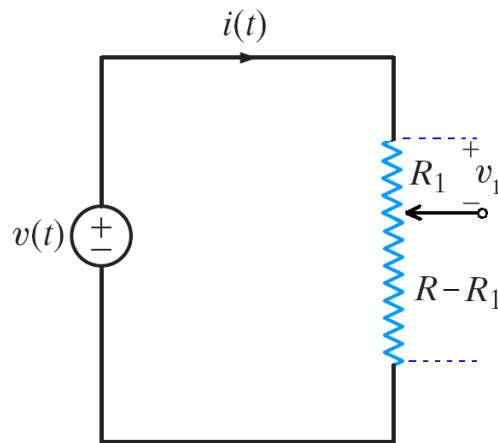


Figure 11. Voltage divider based on a potentiometer.

The Thermistor

The *thermistor* is a resistor with resistance R_T that is a function of temperature. A thermistor can be used as a temperature sensor. The table below is obtained from the data sheet for the thermistor that you will use in the lab. It shows the resistance (in Ω) of the thermistor for different values of temperature (in $^{\circ}\text{C}$). For example, the thermistor has a $10\text{k}\Omega$ resistance at 25°C . The thermistor's resistance decreases as its temperature increases.

$$\begin{bmatrix} T \\ R_T \end{bmatrix} = \begin{bmatrix} -25 & 0 & 25 & 50 & 75 & 100 & 150 & 204 \\ 133300 & 32960 & 10000 & 3577 & 1469 & 671.2 & 182 & 59.9 \end{bmatrix}$$

Figure 12 shows a plot (in red) for the above thermistor data. Exponential function regression, of the form $R_T = A.B^T$, can be used to arrive at an analytical expression that approximates the thermistor's R - T behavior (your scientific calculator, such as the TI-83, can be used for regression analysis). Regression analysis was performed on the above data and led to the following expression:

$$R_T = 27948(0.9674)^T \quad (6)$$

The plot for Equation (6) is shown in Figure 12 (in blue color); it is a fair fit for the thermistor data as long as $T > 0$.

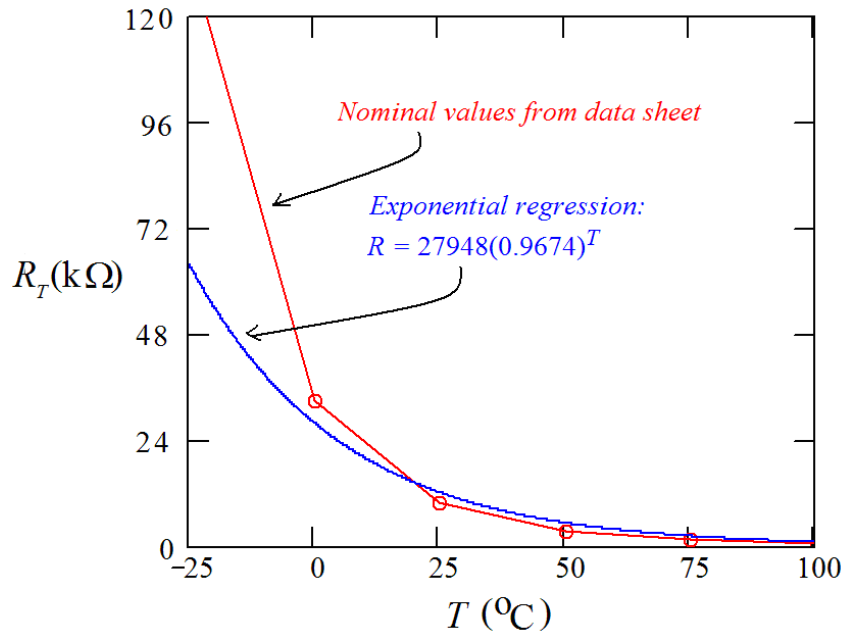


Figure 12. A plot of thermistor resistance versus temperature.

A thermistor-based temperature sensor circuit is depicted in Figure 13. This circuit is essentially a voltage divider. The thermistor's resistance is R_T . The sensor circuit *encodes* the temperature as a voltage v_T , across the thermistor.

This voltage can then be further processed (say, by a microprocessor) to obtain the temperature value; T is obtained by solving the equations:

$$R_T = 27948(0.9674)^T \text{ and } v_T = \frac{5R_T}{10,000 + R_T} \quad (7)$$

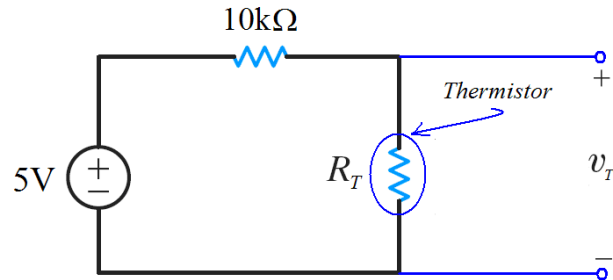


Figure 13. Thermistor-based temperature sensor.

It is left as a pre-lab exercise for the student to derive the following analytical expression for T as a function of v_T (temperature sensor model):

$$T = 30.17 \left[10.24 - \ln \left(\frac{10000v_T}{5 - v_T} \right) \right] \quad (8)$$