

ECE 3300
Electrical Circuits: Laboratory
Experiment 5
Circuit Laws

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In this experiment, the student will verify three important circuit laws: Superposition Law, Kirchhoff's Voltage Law and Kirchhoff's Current Law.

Learning Objectives

By the end of this unit, the student should be able to build simple resistive circuits with independent sources and employ laboratory measurements to:

- Obtain circuit voltages and currents employing the Superposition Law.
- Verify Kirchhoff's Voltage Law.
- Verify Kirchhoff's Current Law.

The Superposition Law

The *Superposition Law* applies to circuits consisting of (only) *linear* devices, driven by two or more sources. Resistive circuits are examples of linear circuits. The Superposition Law requires that the total effect of several causes acting simultaneously be equal to the sum of the effects of the individual causes acting one at a time.

The *Superposition Law* may be restated in the context of a linear circuit with two or more independent sources, as follows: *The voltage across (current through) an element may be obtained by algebraically adding all the individual voltages (currents) caused by each source acting alone, with all other sources deactivated.*

A voltage source is deactivated by setting its value to zero or replacing it by a short circuit. A current source is deactivated by setting its value to zero or replacing it by an open circuit.

The following example illustrates the application of the Superposition Law to solve for v_2 , the voltage across resistor R_2 , in the circuit in Figure 1. By

superposition, v_2 is equal to the voltage across R_2 with v_{s2} set to zero added to the voltage across R_2 with v_{s1} set to zero; mathematically, we may write:

$$v_2 = v_2|_{v_{s2}=0} + v_2|_{v_{s1}=0} \quad (1)$$

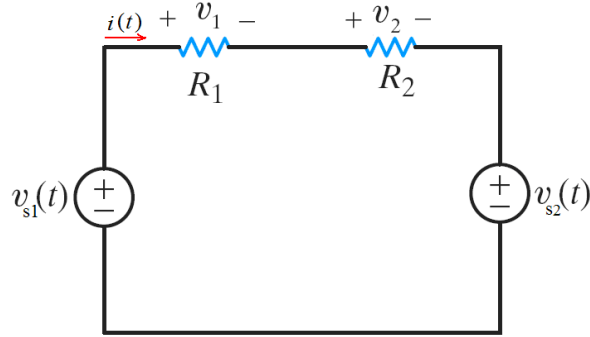


Figure 1. A resistive circuit driven by two voltage sources.

The application of the Superposition Law, as implied by Equation (1), is achieved by analyzing two separate sub-circuits. These circuits are shown in Figure 2.

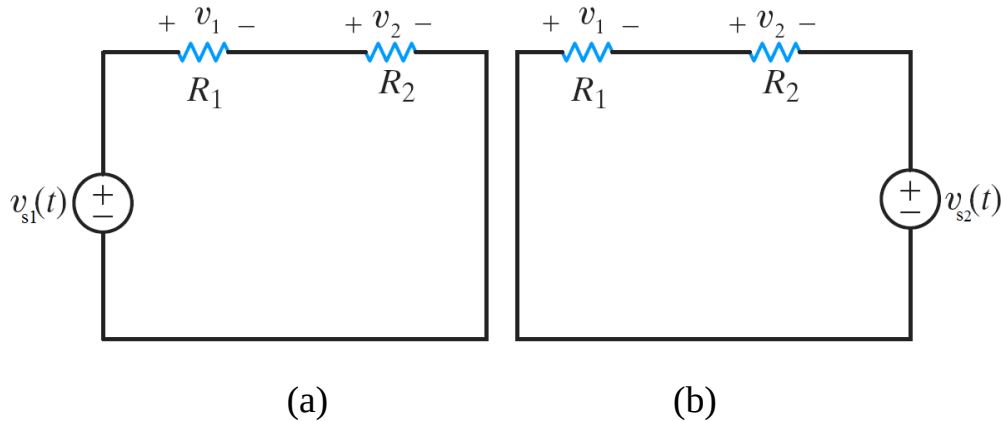


Figure 2. Superposition sub-circuits for the circuit in Figure 1: (a) $v_{s2} = 0$ and (b) $v_{s1} = 0$.

Both circuits in Figure 2, are voltage dividers. Therefore, with the aid of the voltage division rule, Equation (1) can be written as:

$$v_2 = v_2|_{v_{s2}=0} + v_2|_{v_{s1}=0} = \frac{R_2}{R_1 + R_2} v_{s1} - \frac{R_2}{R_1 + R_2} v_{s2} = \frac{R_2}{R_1 + R_2} (v_{s1} - v_{s2}) \quad (2)$$

It is important to note the term “*algebraically*” in the earlier statement of the Superposition Law; this is what gives rise to the negative sign in Equation (2).

Superposition may also be used to solve for the current $i(t)$ in the circuit in Figure 1. Employing Ohm’s Law in the circuits in Figures 2(a) and (b), one can derive the following expression for $i(t)$:

$$i(t) = i(t)|_{v_{s2}=0} + i(t)|_{v_{s1}=0} = \frac{v_{s1}}{R_1 + R_2} - \frac{v_{s2}}{R_1 + R_2} = \frac{v_{s1} - v_{s2}}{R_1 + R_2} \quad (3)$$

Kirchhoff’s Voltage Law

Kirchhoff’s Voltage Law (KVL) states that the *algebraic* sum of the voltages around any *closed path* in a circuit is identically zero for all time.

Consider the circuit in Figure 3. The circuit consists of a single mesh (loop). We can translate KVL for this circuit into a mathematical expression. First, choose a *path direction* to follow. In Figure 3, the loop direction (shown in red) is chosen to be in a clock-wise direction. Now, add (algebraically) all voltage drops around the loop and set the sum to zero. If the loop path enters the positive (labeled) terminal of a component, then the component voltage drop is set positive; otherwise, a negative sign is attached to the voltage. With this in mind, the KVL for the circuit in Figure 3 is expressed as:

$$-v_{s1} + v_1 + v_2 + v_{s2} = 0 \quad (4)$$

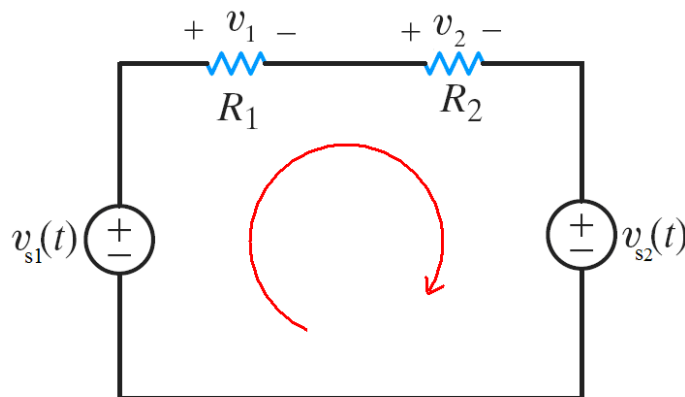


Figure 3. A one-mesh resistive circuit used to illustrate Kirchhoff’s Voltage Law.

KVL can be verified experimentally by using a voltmeter to measure the voltages across all circuit components, and then substituting the measured values in Equation (4). The voltmeter placement must be consistent with the reference polarity placed across the components as indicated in the circuit diagram. Figure 4 depicts the proper way to place the voltmeter in the circuit of Figure 3, so the voltage value is obtained with the correct sign.

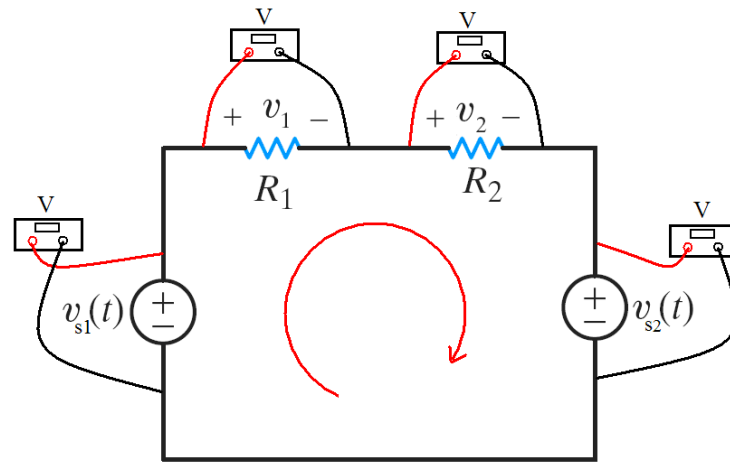


Figure 4. Proper voltmeter placement for the verification of Kirchhoff's Voltage Law.

If one, or more, *time dependent* voltage source is present in a circuit, then a scope is needed to display and measure circuit voltages. However, the scope (as opposed to a voltmeter) can't be used to measure the voltage of a floating component. This issue was discussed earlier in the Scope Unit (refer to the section entitled "Connecting a Voltage Signal.") For example, the scope's probe must not be connected as shown in Figure 5 when measuring the voltage across R_2 ; this leads to shorting the v_{s2} source!

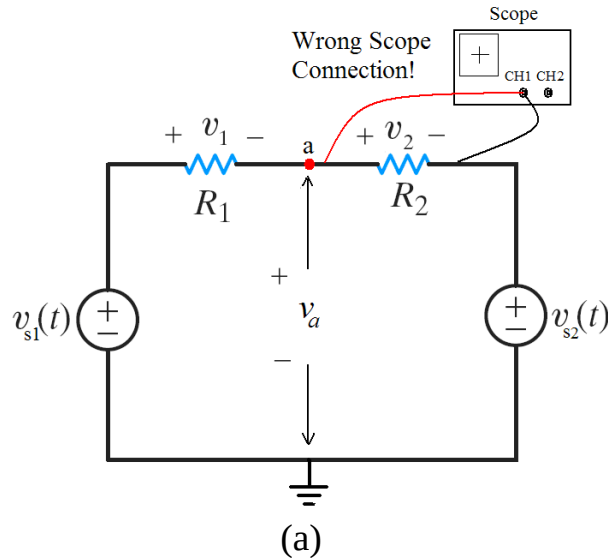


Figure 5. Wrong scope connection to a circuit.

Figure 6 depicts the proper way of connecting a scope to a circuit to measure a voltage signal. In this case, the voltage signal being measured is the node voltage v_a ; the voltage drop at node a with respect to the ground node.

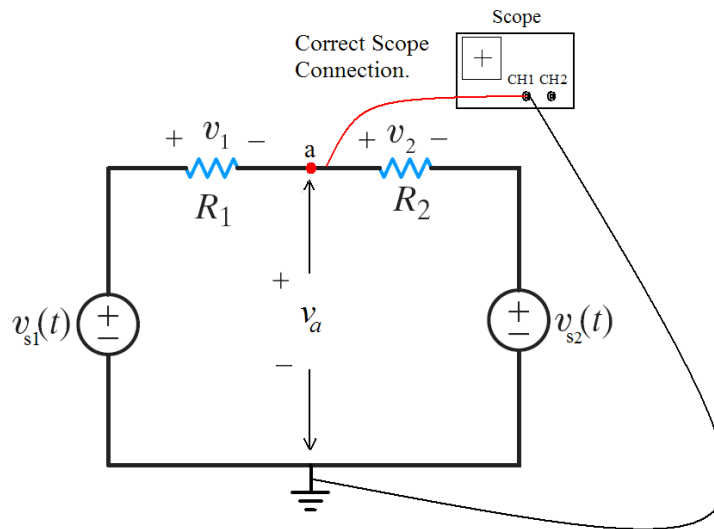


Figure 6. Correct scope connection to a circuit to measure node voltage v_a .

But how do we measure the voltage v_2 (the voltage across R_2) using the scope? Well, the trick is to measure v_2 as the difference of two voltage signals. These signals can be acquired by the scope on Channels 1 and 2, respectively. Then, the scope's Math Function can be used to compute and display the difference between Channel 1 and Channel 2.

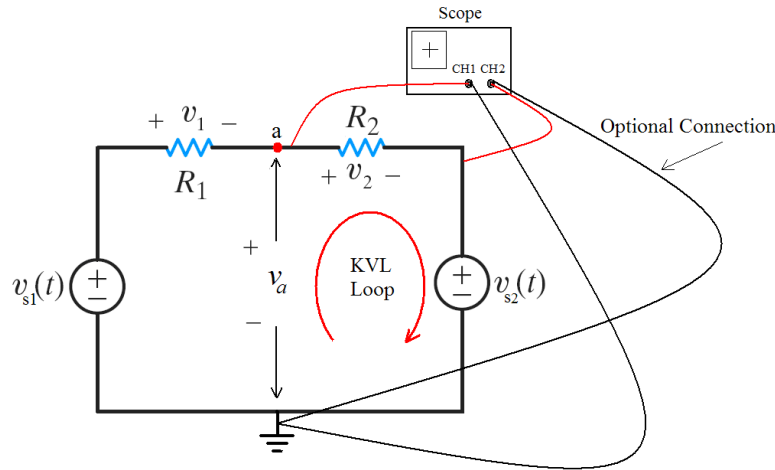


Figure 7. The scope is shown to acquire the voltage signals v_a and v_{s2} on Channels 1 and 2, respectively.

Figure 7 depicts a scope acquiring v_a on Channel 1 and acquiring v_{s2} on Channel 2. The figure also shows a voltage drop loop. Applying KVL at this loop gives: $-v_a + v_2 + v_{s2} = 0$, which leads to $v_2 = v_a - v_{s2}$. Therefore, v_2 can be displayed on the scope as Channel 1 minus Channel 2.

Kirchhoff's Current Law

Kirchhoff's Current Law (KCL) states that the algebraic sum of the currents entering any node is identically zero for all instants of time.

Consider the circuit in Figure 8. The circuit consists of four nodes (including the ground node). We can translate KCL at the center node (the node connecting the three resistors together) into a mathematical expression. First, all currents flowing into the node must be labeled. Then, all currents are added (algebraically) and the sum is set to zero. Applying KCL at the center node results in:

$$i_1 + i_2 + i_3 = 0 \quad (5)$$

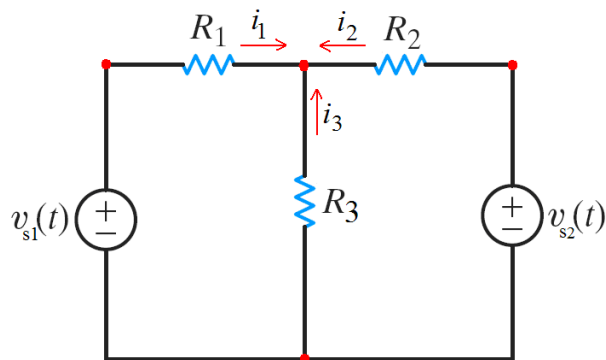


Figure 8. Resistive circuit used to illustrate Kirchhoff's Current Law.

KCL can be verified experimentally by using an ammeter to measure the currents entering a given node, and then substituting the measured values in Equation (5). The ammeter placement must be consistent with the reference direction of the currents as indicated in the circuit diagram. Figure 9 shows the proper way to place the ammeter in the circuit of Figure 8 so you obtain the current value with the correct sign.

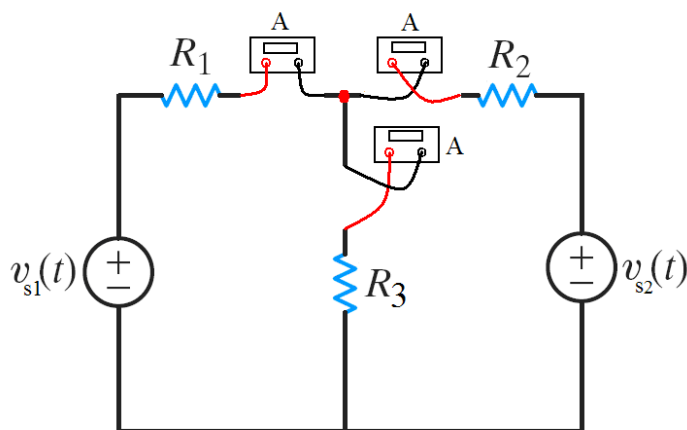


Figure 9. Ammeter placement for verifying Kirchhoff's Current Law for the circuit in Figure 8.