

ECE 3300
Electrical Circuits: Laboratory
Experiment 6 Pre-Reading
Circuit Theorems

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In this experiment, the student will verify two important circuit theorems: Thevenin's Theorem and the Maximum Power Transfer Theorem.

Learning Objectives

By the end of this unit, the student should be able to build simple resistive circuits, powered by independent sources, and employ laboratory measurements to:

- Experimentally obtain the Thevenin equivalent circuit.
- Employ source transformation to the Thevenin circuit to obtain the Norton equivalent circuit.
- Experimentally verify the Maximum Power Transfer Theorem.

Thevenin's Theorem

Back in 1883, a French engineer by the name M. L. Thevenin had developed a theorem that allows a portion of a complex resistive circuit to be reduced to a simple circuit consisting of a voltage source in series with a resistor. This simple circuit is called the *Thevenin equivalent circuit*. The following is a formal statement of Thevenin's Theorem:

For any circuit of resistive elements and power sources (current and/or voltage) with an identified terminal pair $a-b$, the circuit can be replaced by a series combination of an ideal voltage source v_t and a resistor R_t . Here, the Thevenin voltage v_t is the voltage across $a-b$ (or the open-circuit voltage, v_{oc}) and R_t (the Thevenin resistance) is the ratio of the open-circuit voltage to the short-circuit current (i_{sc}) at the terminal pair $a-b$.

Once the terminal pair $a-b$ has been identified, the *Thevenin equivalent circuit* can be easily obtained, experimentally, by making one voltage measurement and one current measurement. Consider the general resistive circuit depicted in Figure 1. We will assume, without loss of generality, that the circuit has independent DC power sources. The circuit consists of two sub-circuits: A and

B. The terminal pair a - b represents any two independent nodes in the circuit such that Circuit *A* has at least one independent power source.

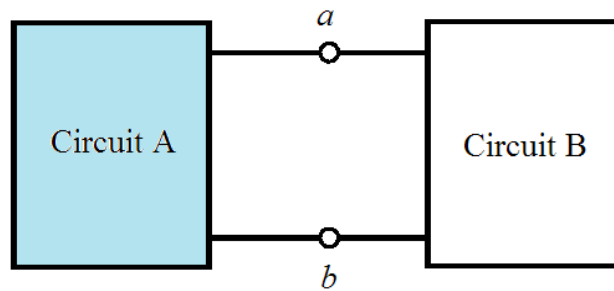


Figure 1. A general resistive circuit with independent power sources.
The circuit consists of two sub-circuits, *A* and *B*.

According to Thevenin's Theorem, Circuit *A* can be reduced to a voltage source in series with a single resistor. Thus, the circuit in Figure 1 is reduced to the circuit in Figure 2. The operation of Circuit *B* (as far as values of currents and voltages) in both the original and the reduced circuit is not affected. In some cases, Circuit *B* is simply a load resistor R_L .

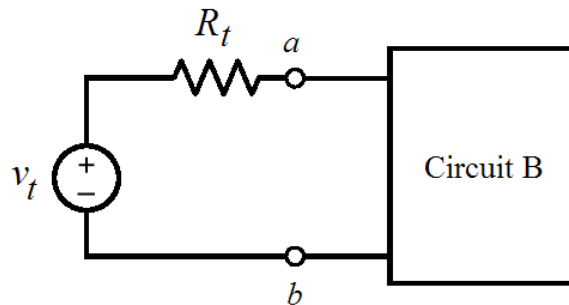


Figure 2. A circuit equivalent to the circuit in Figure 1. Here, Circuit *A* has been replaced by its Thevenin equivalent circuit.

Experimentally, the Thevenin equivalent circuit can be obtained by employing the following method (refer to Figure 3):

- Disconnect Circuit *A* from Circuit *B*.
- Use a voltmeter to measure the voltage drop at node a with respect to node b . This is the open-circuit voltage, v_{oc} (recall that, ideally, the voltmeter is modeled as an open circuit).
- Set v_t equal to v_{oc} .

- Connect an ammeter across terminal a - b . The measured current is the short-circuit current, i_{sc} (recall that, ideally, the ammeter is modeled as a short circuit).
- Compute the value of the Thevenin resistor: $R_t = v_{oc}/i_{sc}$.

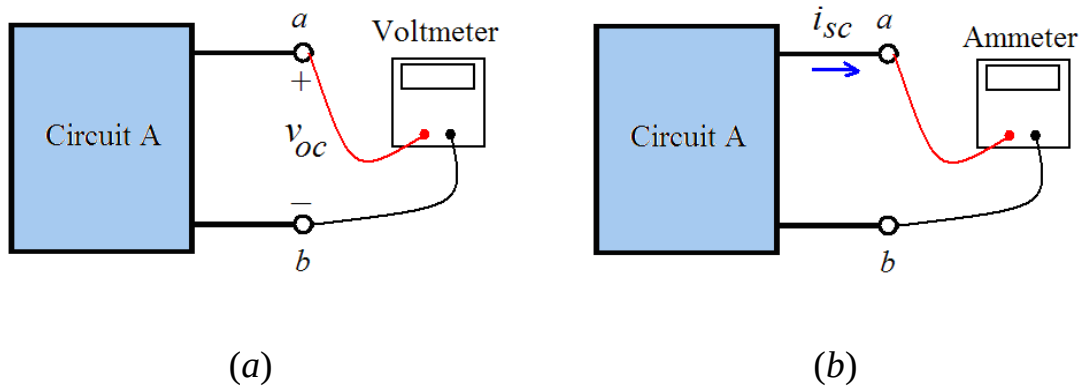


Figure 3. Experimental setup for measuring (a) the open-circuit voltage and (b) the short-circuit current at terminal a - b .

The following is a Multisim example that illustrates how to use simulation to verify Thevenin's Theorem. Consider the circuit in Figure 4. The circuit consists of two sections, A and B: Circuit A drives a load resistor R_L (Circuit B). We are interested in obtaining the Thevenin equivalent circuit for Circuit A (the portion of the circuit to the left of terminal pair a - b).

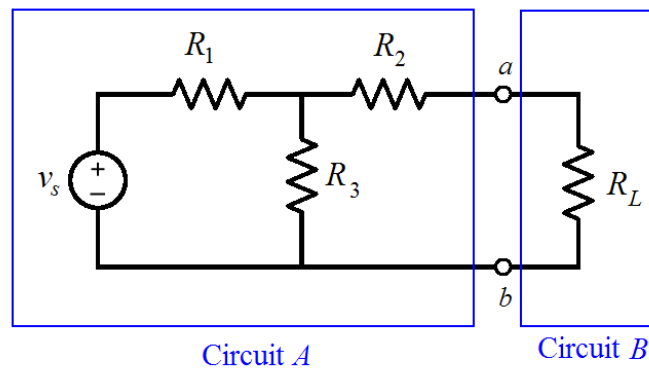


Figure 4. A resistive circuit example used to illustrate Thevenin's Theorem.

Let us assume that $v_s = 12\text{V}$, $R_1 = 1\text{k}\Omega$, $R_2 = 1\text{k}\Omega$, $R_3 = 1\text{k}\Omega$ and $R_L = 1\text{k}\Omega$. Figure 5 depicts the Multisim analysis results for the voltage across R_L . Figure 6a and 6b show the Multisim results for v_{oc} and i_{sc} , respectively, for Circuit A (note how the load resistor has been removed).

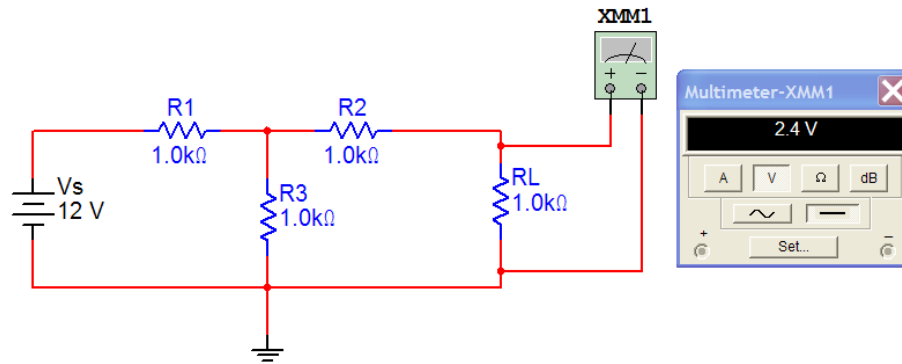
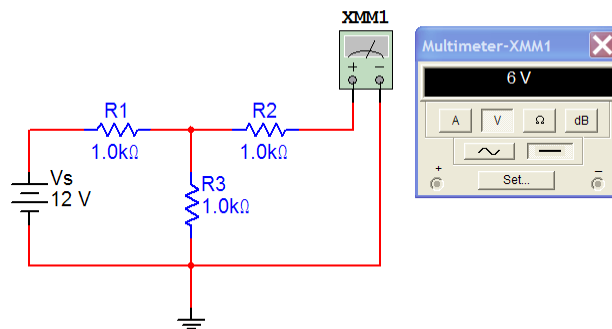
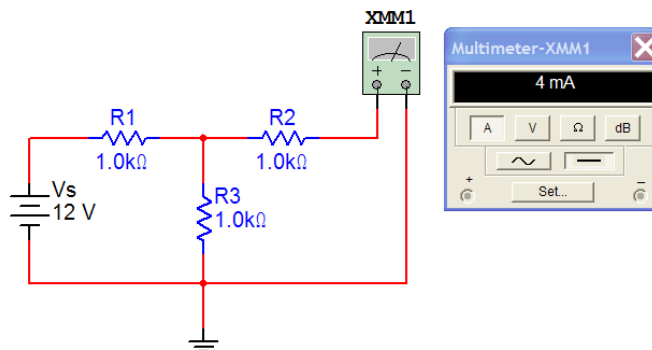


Figure 5. Multisim result for the voltage across the load resistor.



(a)



(b)

Figure 6. (a) Multisim results for the open-circuit voltage and (b) the short circuit current.

Figure 7 shows the original circuit (in Figure 4), but with the Thevenin equivalent circuit. The Thevenin voltage source is $v_t = v_{oc} = 6\text{V}$. The Thevenin resistance has the value $R_t = v_{oc}/i_{sc} = 6/0.004 = 1.5\text{k}\Omega$. Note that the voltage

across the load resistor in Figure 7 is equal to that in the original circuit (refer to Figure 5).

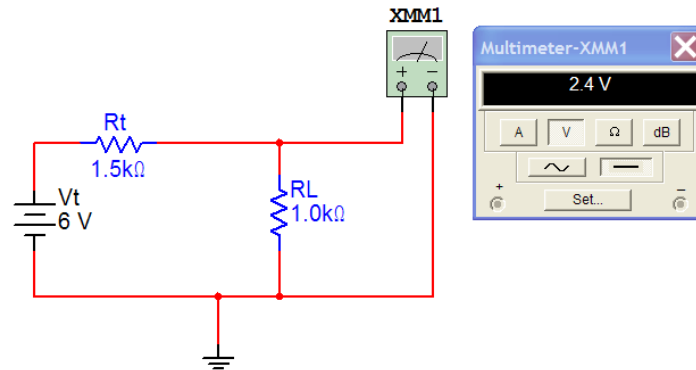


Figure 7. Multisim result for the load resistor voltage employing the Thevenin equivalent circuit.

Source Transformation

Consider the simple circuit consisting of an independent current source, $i_s(t)$, in parallel with a resistor R (refer to Figure 8). Let us determine the Thevenin circuit at terminal pair $a-b$. The open-circuit voltage is the voltage across the resistor, which (by Ohm's Law) is given by $v_{oc} = Ri_s$. The short circuit current, i_{sc} , is equal to i_s as can be shown by applying the current division rule (refer to Figure 9). Then, the Thevenin resistor is $R_t = v_{oc}/i_{sc} = Ri_s/i_{sc} = Ri_s/i_s = R$.

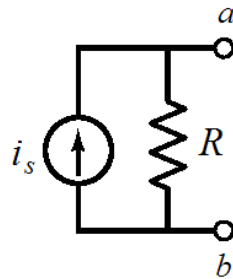


Figure 8. A simple circuit with an independent current source in parallel with a resistor.

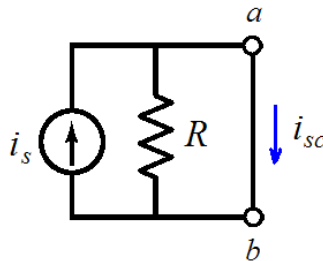


Figure 9. Circuit for computing the short-circuit current.

The complete Thevenin equivalent circuit is shown in Figure 10. The results are summarized as depicted in Figure 11. The results suggest that one can transform a current source, i_s , in parallel with a resistor R into a voltage source, $v_s = Ri_s$, in series with a resistor R . Alternatively, one can transform a voltage source, v_s , in series with a resistor R into a current source, $i_s = v_s/R$, in parallel with the resistor R (note the polarity of the power sources). This last statement suggests that (Norton's Theorem): **A resistive circuit with power sources can always be transformed into a simple circuit consisting of a current source in parallel with a single resistor.**

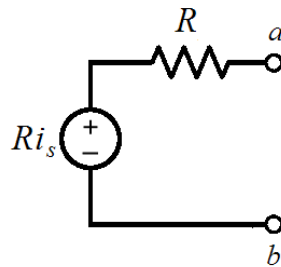


Figure 10. The Thevenin equivalent circuit for the circuit in Figure 8.

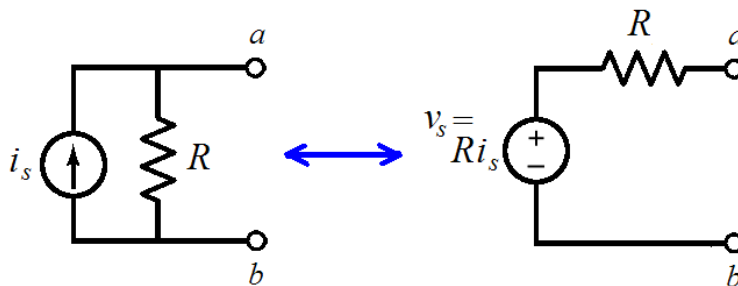


Figure 11. Source transformation.

The Maximum Power Transfer Theorem

Consider a resistive circuit powering a load resistor R_L (refer to Figure 12a). We wish to find the value of the load resistor such that maximum power is delivered to it; here, it is assumed that the resistive circuit has components with fixed values.

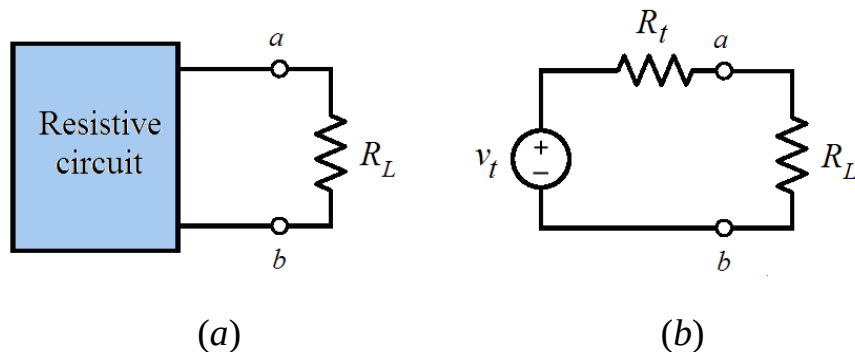


Figure 12. (a) A general resistive circuit and (b) its Thevenin equivalent circuit.

Figure 12b shows the load resistor connected to the output of the Thevenin equivalent circuit. **The Maximum Power Transfer Theorem states that the power dissipated in R_L is maximized when $R_L = R_t$.** The theorem can be proved by noting that the power, p , dissipated in R_L is given by:

$$p = v_{R_L} i_{R_L} = i_{R_L}^2 R_L = \left(\frac{v_t}{R_L + R_t} \right)^2 R_L \quad (1)$$

The function in Equation (1) is maximized when $R_L = R_t$, giving the maximum power:

$$p_{\max} = \frac{v_t^2}{4R_t} \quad (2)$$

The normalized power, $p/p_{\max}$, is plotted against R_L/R_t in Figure 13.

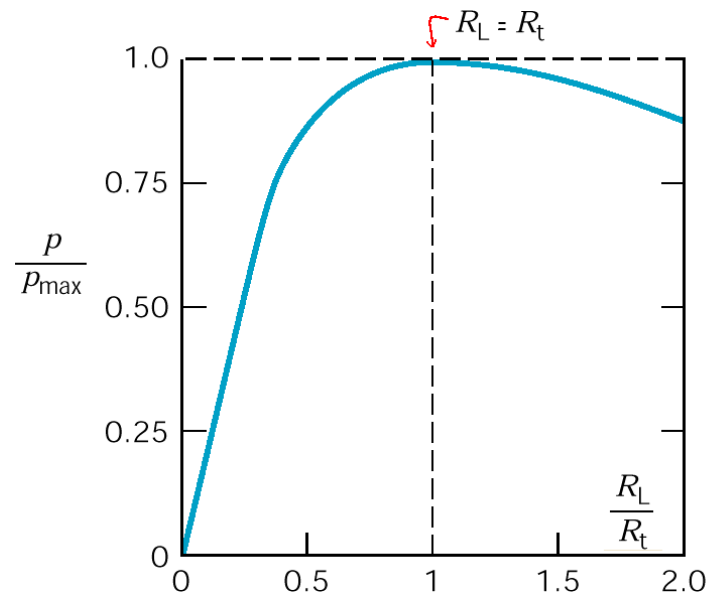


Figure 13. Plot of the normalized power dissipated in the load resistor R_L .