

Properties of the unit-impulse function, $\delta(t)$

1.) $0 \cdot \delta(t) = 0$

2.) $\delta(t) = \delta(-t)$

3.) $\delta(t - a) = \delta(a - t)$

4.) $f(t) \delta(t) = f(0) \delta(t)$

5.) $f(t) \delta(t - a) = f(a) \delta(t - a)$

6.) $\int_{-\infty}^{+\infty} f(t) \delta(t) dt = f(0)$

7.) $\int_{-\infty}^{+\infty} f(t) \delta(t - a) dt = f(a)$

8.) $\int_{-\infty}^{+\infty} f(\tau) \delta(t - \tau) d\tau = f(t)$

9.) $\delta(at) = \frac{1}{|a|} \delta(t)$ (where $a \neq 0$)

10.) $\int_{-\infty}^t \delta(\tau) d\tau = u(t)$

11.) $\delta(t) = \frac{du(t)}{dt} = \dot{u}(t)$. Also, $\dot{u}(t - a) = \delta(t - a)$

12.) $\int_{-\infty}^{+\infty} f(t) \dot{\delta}(t) dt = -\dot{f}(0) = -\left. \frac{df(t)}{dt} \right|_{t=0}$

In general, $\int_{-\infty}^{+\infty} f(t) \delta^{(n)}(t - t_0) dt = (-1)^n f^{(n)}(t_0)$

13.) $t \dot{\delta}(t) = -\delta(t)$, and $t^n \delta^{(n)}(t) = (-1)^n n! \delta(t)$

14.) $f(t) \dot{\delta}(t) = f(0) \dot{\delta}(t) - \dot{f}(0) \delta(t)$.

15.) $f(t) \ddot{\delta}(t) = f(0) \ddot{\delta}(t) - 2\dot{f}(0) \dot{\delta}(t) + \ddot{f}(0) \delta(t)$

16.) $\delta(g(t)) = \sum_{i=1}^m \frac{\delta(t-t_i)}{|g'(t_i)|}$, t_i are the real roots of $g(t) = 0$.