

Consider the complex Fourier coefficients for a **periodic signal** $f(t)$:

$$D_n \quad (n = 0, \pm 1, \pm 2, \dots)$$

$$D_n = |D_n| e^{j\angle D_n} \quad (D_n \text{ is generally complex})$$

$$D_n = \text{Re}\{D_n\} + j \text{Im}\{D_n\}$$

D_0 is real

If the signal* $f(t) = f_{\text{even}}(t) + f_{\text{odd}}(t)$ is **real**, then the exponential Fourier series coefficients have the following properties:

1. $D_n = D_{-n}^* \Rightarrow |D_n| = |D_{-n}|$ and $\angle D_n = -\angle D_{-n}$
2. $f_{\text{even}}(t) \Leftrightarrow \text{Re}\{D_n\}$ and $f_{\text{odd}}(t) \Leftrightarrow j \text{Im}\{D_n\}$
3. $\text{Re}\{D_n\}$ and $|D_n|$ are even
4. $\text{Im}\{D_n\}$ and $\angle D_n$ are odd
5. If $f(t)$ is even, then $\text{Im}\{D_n\} = 0$
6. If $f(t)$ is odd, then $\text{Re}\{D_n\} = 0$

If $f(t)$ is a power signal, then (according to Parseval's theorem):

$$\text{Signal Power} = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} |f(t)|^2 dt = C_0^2 + \frac{1}{2} \sum_{n=1}^{\infty} C_n^2 = D_0^2 + 2 \sum_{n=1}^{\infty} D_n^2$$

*Note: a real function $f(t)$ can always be written as the sum of an odd function and an even function: $f(t) = f_{\text{even}}(t) + f_{\text{odd}}(t)$ with

$$f_{\text{even}}(t) = \frac{1}{2} [f(t) + f(-t)]$$

$$f_{\text{odd}}(t) = \frac{1}{2} [f(t) - f(-t)]$$