

Properties of Convolution

1. Commutative: $f_1(t) * f_2(t) = f_2(t) * f_1(t)$

$$\text{i.e., } \int_{-\infty}^{+\infty} f_1(\tau) f_2(t - \tau) d\tau = \int_{-\infty}^{+\infty} f_2(\tau) f_1(t - \tau) d\tau$$

2. Associative: $f_1 * (f_2 * f_3) = (f_1 * f_2) * f_3 = (f_1 * f_3) * f_2$

3. Distributive: $f_1 * (f_2 \pm f_3) = f_1 * f_2 \pm f_1 * f_3$

4. Time shift: Given, $f_1(t) * f_2(t) = g(t)$, then

$$f_1(t - a) * f_2(t) = f_1(t) * f_2(t - a) = g(t - a)$$

5. Multiplication by a constant:

$$\text{Given, } f_1(t) * f_2(t) = g(t), \text{ then } b f_1(t) * f_2(t) = b g(t)$$

6. Convolution with the unit-impulse, $\delta(t)$:

$$f(t) * \delta(t) = \int_{-\infty}^{+\infty} f(\tau) \delta(t - \tau) d\tau = f(t) \int_{-\infty}^{+\infty} \delta(t - \tau) d\tau = f(t)$$