Expressing the power of a sinusoid as a function of harmonic sinusoids

n even:

$$\cos^n x = \frac{1}{2^n}\binom{n}{\frac{n}{2}} + \frac{2}{2^n}\sum_{k=0}^{\frac{n}{2}-1}\binom{n}{k}\cos[(n-2k)x]$$

$$\sin^n x = \frac{1}{2^n}\binom{n}{\frac{n}{2}} + \frac{2}{2^n}\sum_{k=0}^{\frac{n}{2}-1}(-1)^{\left(\frac{n}{2}-k\right)}\binom{n}{k}\cos[(n-2k)x]$$

n odd:

$$\cos^n x = \frac{2}{2^n}\sum_{k=0}^{\frac{n-1}{2}}\binom{n}{k}\cos[(n-2k)x]$$

$$\sin^n x = \frac{2}{2^n}\sum_{k=0}^{\frac{n-1}{2}}(-1)^{\left(\frac{n-1}{2}-k\right)}\binom{n}{k}\sin[(n-2k)x]$$

With the binomial coefficient:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \quad \text{for } 0 \le k \le n$$

Examples:

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\sin^3 x = \frac{3\sin x - \sin 3x}{4}$$

$$\sin^4 x = \frac{3-4\cos 2x + \cos 4x}{8}$$

$$\sin^5 x = \frac{10\sin x - 5\sin 3x + \sin 5x}{16}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\cos^3 x = \frac{3\cos x + \cos 3x}{4}$$

$$\cos^4 x = \frac{3+4\cos 2x + \cos 4x}{8}$$

$$\cos^5 x = \frac{10\cos x + 5\cos 3x + \cos 5x}{16}$$