

Experiment 3

Analog Integration and Application to Oscillator Design

Systems and Signals Laboratory

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Electronic components required for this experiment

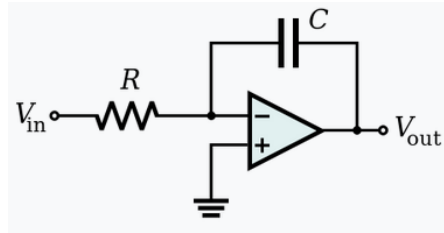
- LM741 standard operational amplifier (1)
- LF351 standard operational amplifier (3)
- OPA277 precision operational amplifier (1)
- 5.1k Ω (1/4 Watt, 5%) resistor (1)
- 10k Ω (1/4 Watt, 1%) resistor (4)
- 1M Ω (1/4 Watt, 5%) resistor (1)
- 10k Ω pot
- 20k Ω pot
- 0.1 μ F (polystyrene film, 10%) capacitor (2)
- Switch [SPST, OFF(ON)]

1. Pre-lab

Read the following material. Solve the problems presented below. Submit your pre-lab report to the lab instructor at the beginning of the next lab session.

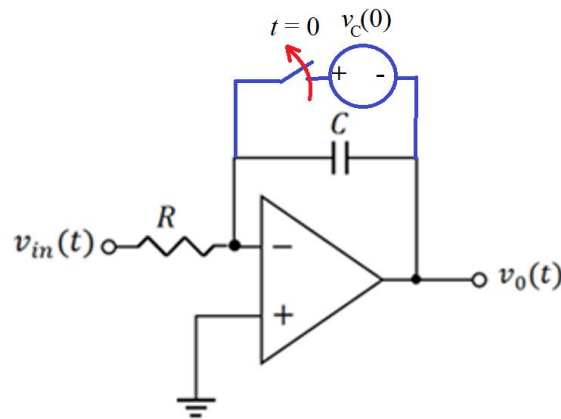
Analog Integration

You have first encountered the following (analog) inverting integrator circuit in your basic circuit analysis course.

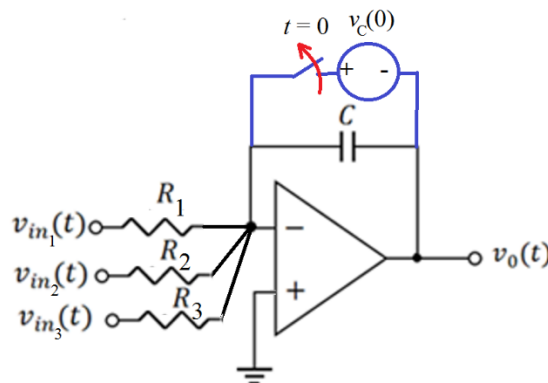


Problem 1. Let the initial voltage across the capacitor be $v_c(0^-)$, with the specified polarity as shown in the figure below. This also sets the initial op amp output voltage to $v_o(0^-) = -v_c(0^-)$. Show that, after the switch opens, the output voltage $v_o(t)$ is given by

$$v_o(t) = \frac{-1}{RC} \int_{0^-}^t v_{in}(\tau) d\tau - v_c(0^-)$$



A generalization of the above basic integrator is the inverting summing integrator shown below. This circuit is the basis for the analog computers of the 50's and 60's era.

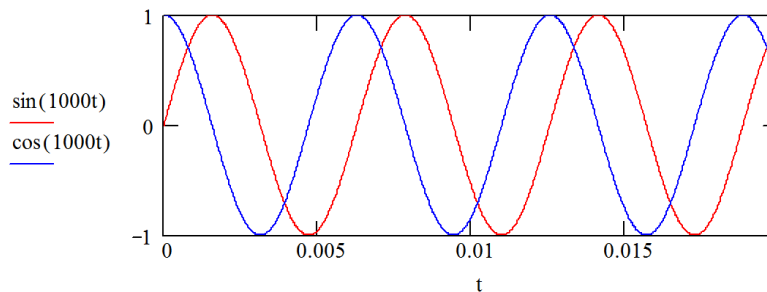


The above circuit has the following input/output relationship,

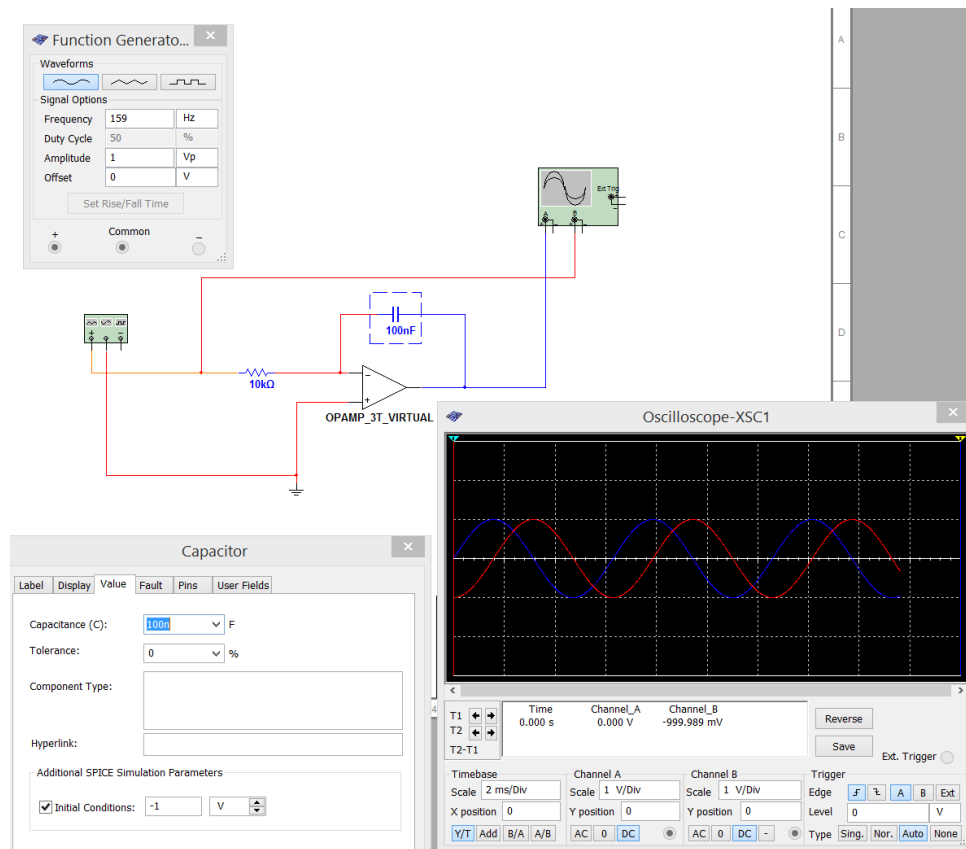
$$v_o(t) = - \int_{0^-}^t \left[\frac{1}{R_1 C} v_{in1}(\tau) + \frac{1}{R_2 C} v_{in2}(\tau) + \frac{1}{R_3 C} v_{in3}(\tau) \right] d\tau - v_c(0^-)$$

Example

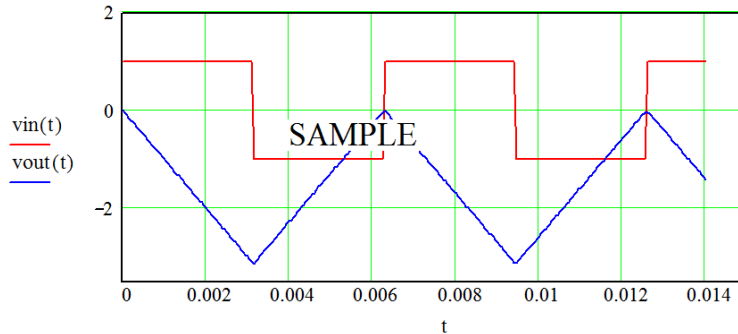
Let us design the inverting integrator to generate an output $v_o(t) = \cos(1000t)$ from the input signal $v_{in}(t) = \sin(1000t)$. The integrator output is given by $v_o(t) = -\frac{1}{RC} \int_0^t \sin(\omega\tau) d\tau + v_o(0) = -\frac{1}{RC\omega} (-\cos(\omega t) + \cos(0)) + v_o(t) = \frac{1}{RC\omega} \cos(\omega t) - \frac{1}{RC\omega} + v_o(0)$. So, if we desire the output to be a zero-offset sinusoid with 1Volt amplitude, we would have to set $RC = \frac{1}{\omega}$ and $v_o(0) = 1$. Since $\omega = 1000$ rad/sec (corresponds to a frequency $f = \frac{\omega}{2\pi} = \frac{1000}{2\pi} \cong 159\text{Hz}$), then $RC = 0.001$. Setting $R = 10\text{k}\Omega$ results in $C = \frac{0.001}{10,000} = 0.1\mu\text{F}$. The capacitor's initial voltage is set to $v_c(0) = -v_o(0) = -1$. The following is a plot depicting the input and output waveforms for the inverting integrator. The waves are 90° out of phase.



The following is a Multisim simulation of the above circuit employing an ideal op-amp:

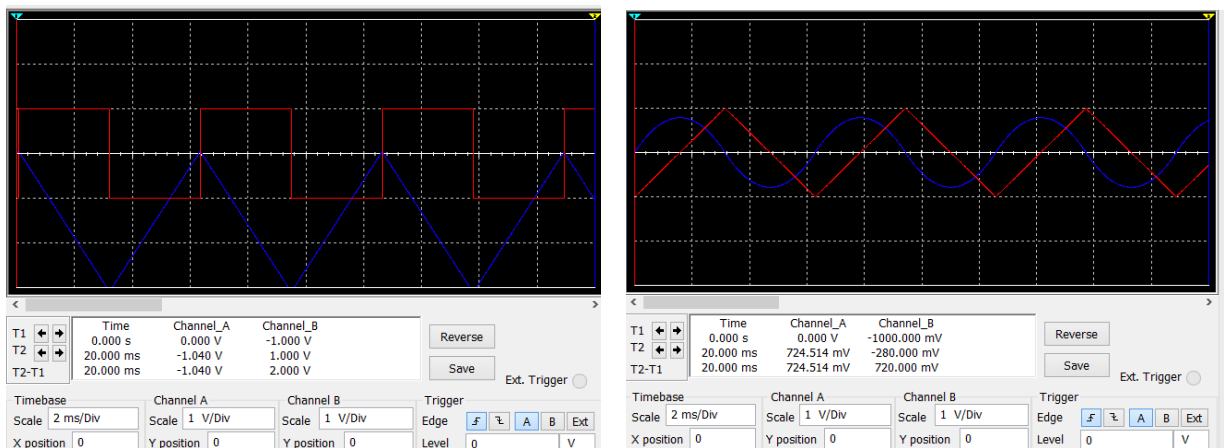


Problem 2. A bipolar square wave with 2Vpp and frequency 159Hz passes through the inverting integrator that we have just designed. Employ Mathcad to generate a single plot showing the input and output waveforms. (For the plot, undo the integrator inversion by multiplying the integrated signal by -1).



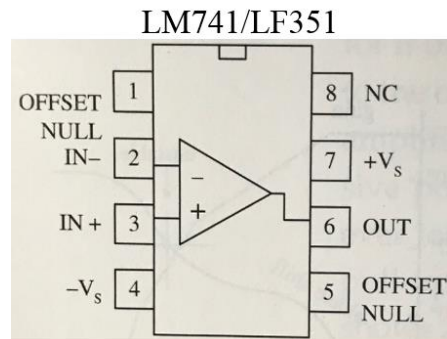
Repeat for a 2Vpp, 159Hz bipolar triangular input wave.

The above Multisim simulation is repeated for a square wave input and a triangular input wave (with $v_o(0) = 0$). The results are shown below.

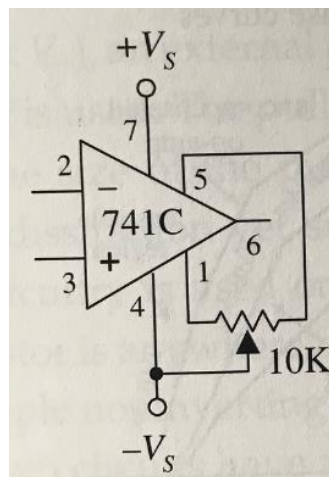


Input Voltage Offset, Input Current Offset and DC Compensation

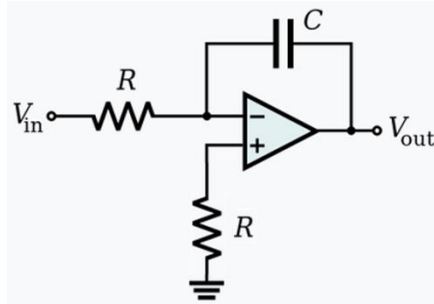
Certain modifications to the above integrator circuit are required in practice since the op amp is not a perfect (ideal) device. For example, the output of a practical op amp (say the LM741) saturates to a value close to one of its power supply voltages, even when both of its input terminals are grounded. This is because the voltage difference between the +ve and -ve terminals of the op amp is not a perfect zero. Practical op amps exhibit a small input offset voltage (about 1mV for the LM741 and 5mV for the wideband LF 351) that causes the output to saturate. This offset voltage is problematic for the integrator circuit: It results in a drift (cumulative summing) of the output voltage. This problem may be overcome by zeroing the input offset voltage. Practical op amp ICs include a pair of voltage offset nulling terminals. These would be pins 1 and 5 for the LM741 and LF351 as depicted in the figure below.



A $10\text{k}\Omega$ potentiometer is placed between these two terminals, while the pot's wiper (center pin) is connected to the negative supply voltage ($-V_s$), as shown in the figure below. The procedure to minimize the offset voltage is to short the op amp input terminals (pin 2 and 3) to ground and then adjust the pot until the output of the op amp approaches zero volts (as monitored on a voltmeter).



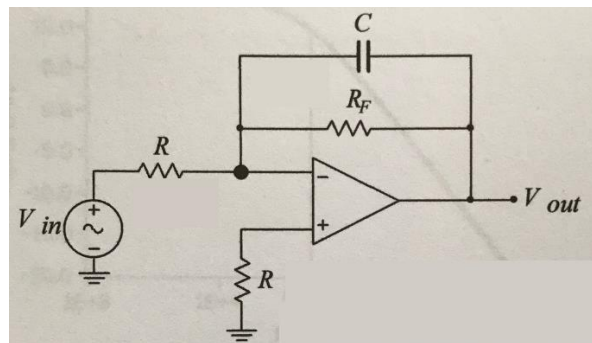
A second problem with some practical bipolar op amps (e.g., LM741) is the relatively large input bias currents (on the order of 80nA). These are the currents that flow into the input terminals of the op amp (ideally, they were assumed to be zero). The presence of the input bias current leads to output voltage errors in amplifiers and integrators. One solution would be to add a resistor R between the non-inverting input and ground as shown in the figure below. Generally speaking, this resistor is set to a value that is equal to the parallel combination of all resistors connected to the inverting input of the op amp. For the basic integrator circuit, the value of this resistor R is selected equal to that of the input resistor.



On the other hand, the JFET LF351 op amp has very, very small input bias currents (on the order of 50pA) and there is no need for the above compensation resistor (but it does not hurt if it is used).

An alternative solution to the input voltage offset and input current bias problems is to use (more expensive) precision op amps, such as the OPA277. This precision op amp has a very small input bias current (on the order of 0.5nA) and an ultra-low input offset voltage (on the order of $\pm 10\mu\text{V}$). For such precision op amps, there is usually no need for the above compensation circuitry.

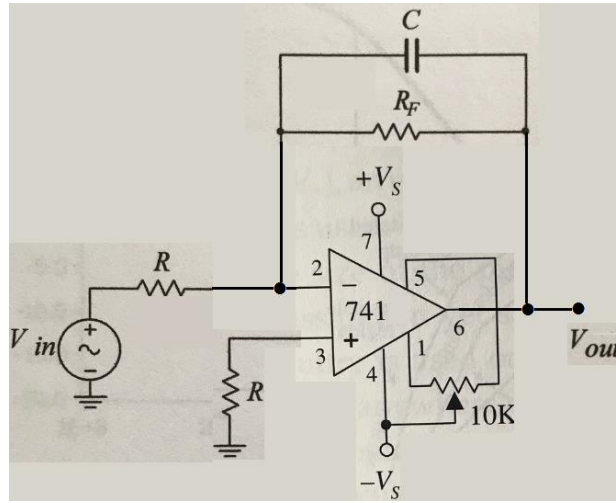
There is also one more problem with practical integrator circuits that one should pay close attention to. It is possible for a tiny DC offset voltage to be present in the input signal. This offset results in drift (cumulative summing) of the output voltage. Small DC offsets (V_{DC}) lead to steady accumulation of charge in the capacitor until the op amp output saturates. One practical solution for eliminating output voltage drift is to provide another feedback path for DC through a high valued resistor (say, $R_F \gg R$) in parallel with the capacitor as shown in the following figure.



This feedback resistor will hold the DC component contribution to the output voltage (through the process of negative feedback) at a value of $-\frac{R_F}{R}V_{DC}$. For example, for $V_{DC} = 0.1\text{mV}$, $R_F = 1\text{M}\Omega$ and $R = 10\text{k}\Omega$ the DC component contribution to the output voltage is controlled at -0.01 Volt . The addition of resistor R_F to the circuit should not affect the integrated signal as long as the frequency of the input signal satisfies, $f \gg \frac{1}{2\pi CR_F}$. **Your turn:** analyze the above op amp circuit (in the s -domain) and show that the circuit is an integrator when this frequency condition is satisfied).

The dielectric absorption (DA) coefficient of a capacitor will also cause drift. A capacitor DA significantly contributes to an op amp's overall settling time; a capacitor with high DA does not completely discharge, and the residual voltage leads to op amp output drift. This problem can be minimized if a capacitor with low DA, such as polypropylene, polystyrene and mica, is used. Electrolytic and tantalum capacitors exhibit high DA and must not be used in op amp based integrators.

In summary, the following figure depicts the complete circuit for a practical inverting integrator, employing the readily available LM741 (or LF351) op amp.



The initial condition $v_o(0)$ can be set by charging the capacitor to that value. If the integrator is to be reset to $v_o(0) = 0$, we would simply briefly short circuit (discharge) the capacitor.

Sine-Wave Oscillator Design and Implementation using Integrator Circuits

In the following, we design a circuit that can generate a voltage $y(t) = A \cos(\omega t)$. This requires a second-order linear circuit with natural radian frequencies $\pm j\omega$ or, equivalently, a characteristic equation $Q(s) = s^2 + \omega^2$. Which, in turn, leads to the second-order linear homogeneous differential equation,

$$\ddot{y} + \omega^2 y = 0 \quad (1)$$

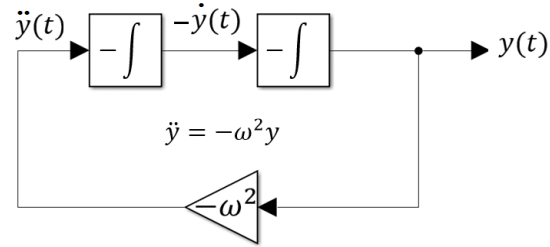
with initial conditions $y(0)$ and $\dot{y}(0)$ to be determined. Rearranging the above equation as

$$\ddot{y} = -\omega^2 y$$

and integrating twice leads to

$$y(t) = \int \int (-\omega^2) y(t) dt$$

This is a relatively simple circuit to build. It requires two inverting integrators and one inverting amplifier, as shown below.



The solution to the differential equation in (1) is given by

$$y(t) = A_1 \cos(\omega t) + A_2 \sin(\omega t)$$

The constants depend on the initial conditions according to

$$y(0) = A_1 \cos(0) + A_2 \sin(0) = A_1$$

and

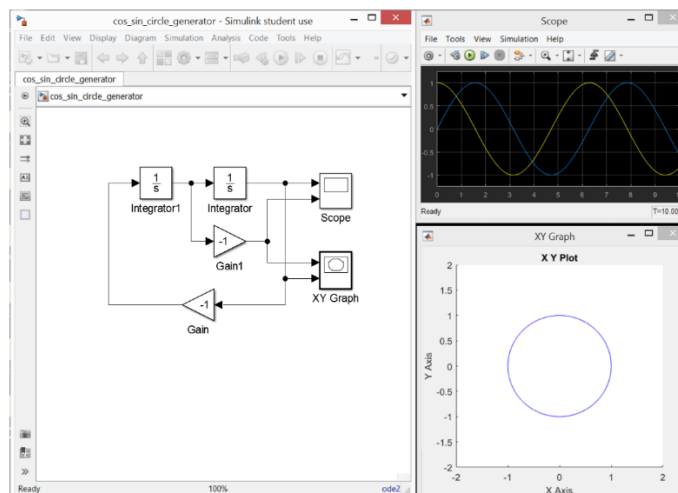
$$\dot{y}(0) = -\omega A_1 \sin(0) + \omega A_2 \cos(0) = \omega A_2$$

leading to the solution $A_1 = y(0)$ and $A_2 = \frac{1}{\omega} \dot{y}(0)$. So, if we want the circuit to output $y(t) = 2\cos(\omega t)$, we would have to set $A_1 = 2$ and $A_2 = 0$. Which requires the initial conditions $y(0) = 2$ and $\dot{y}(0) = 0$. These initial conditions can be set by discharging the capacitor on the first integrator (the one on the left in the above figure) and charging the capacitor of the second integrator to 2V (or, equivalently, briefly connecting a 2V DC source in parallel with the output of the op amp. In the actual oscillator, the noise in the signals can trigger the oscillations.

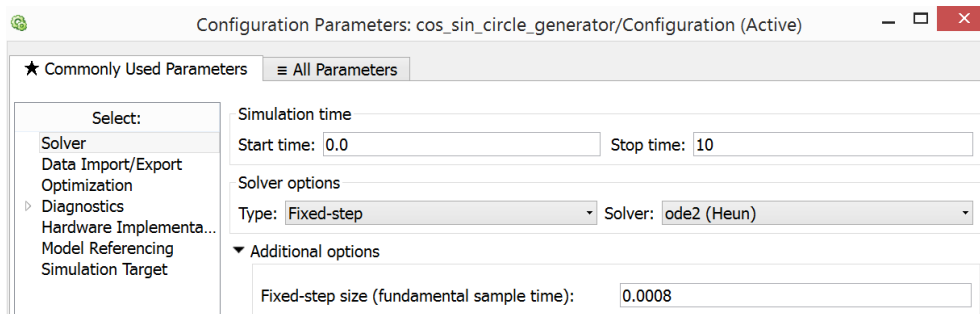
The same circuit can be used to generate a $\sin(t)$ by considering the output of the first integrator. The complete circuit and its Simulink simulation are shown below for $\omega = 1$.

Problem 3. Design a three op amp circuit that realizes the above oscillator.

Problem 4. Employ Matlab's Simulink to verify your oscillator. The following figures depict the Simulink block diagram/program (for $\omega = 1$, or $f = 0.16\text{Hz}$). Note: the $1/s$ integrator blocks in Simulink are non-inverting.



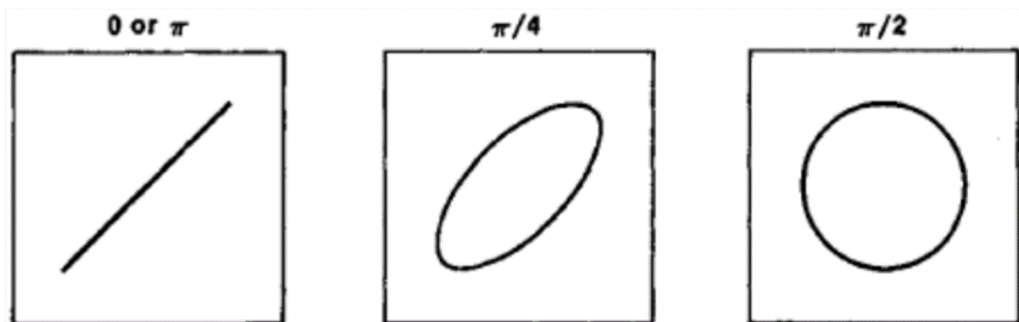
In order to obtain smooth scope traces and XY graphs, you will have to configure the built-in Simulink integration algorithm to use a small integration time step. The following is one possible configuration setting. You might want to try other “solvers” and “step sizes.”



A note on the utilization of the XY plot feature of the scope to estimate the relative phase between two sinusoids.

The student should have already been introduced to Lissajous Figures/Patterns in introductory circuit lab. These figures were first studied by the French scientist J. Lissajous (1822-1880). They can be generated in the lab by feeding two sinusoidal signals $[x(t) = \cos(\omega_1 t + \theta)]$ and $y(t) = \cos(\omega_2 t)]$ as inputs to a two-channel oscilloscope (set to the XY mode). (Watch the video: https://www.youtube.com/shared?ci=mxoti_xPpkU)

The figures can also be easily generated as Matlab or Mathcad. The following are examples of Lissajous Figures depicting XY patterns for three different relative phases among two sinusoids having the same frequency.

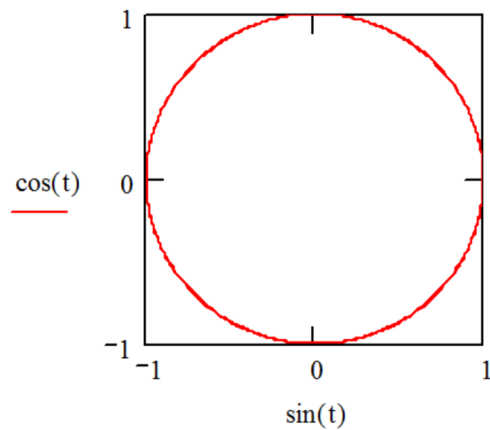


Consider the two signals,

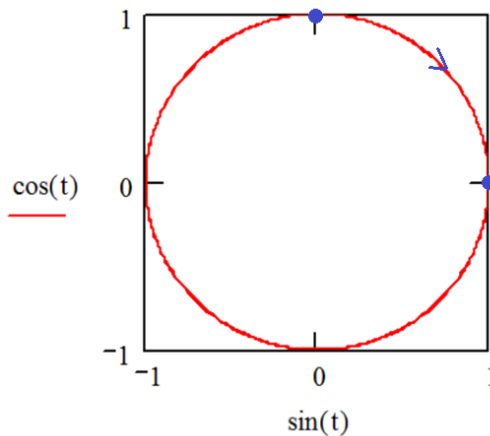
$$x(t) = \cos(\omega t + \theta) \text{ and } y(t) = \cos(\omega t)$$

The shape of the Lissajous Pattern depends on the phase difference θ . For the special case of $\theta = -\frac{\pi}{2}$, we end up plotting $y(t) = \cos(\omega t)$ versus $x(t) = \cos\left(\omega t - \frac{\pi}{2}\right) = \sin(\omega t)$ which leads to a perfect circle of radius one:

$$[x(t)]^2 + [y(t)]^2 = \cos^2(\omega t) + \sin^2(\omega t) = 1$$



In this case, the value of ω does not affect the shape of the figure; it only dictates how fast the point with coordinates $(\sin(\omega t), \cos(\omega t))$ moves around the circle. Without any loss of generality, if we set $\omega = 1$, then for $t = 0$ and $t = \frac{\pi}{2}$ the circle is traced from the point $(\sin(0), \cos(0)) = (0,1)$ to the point $(\sin(\pi/2), \cos(\pi/2)) = (1,0)$. This indicates that the circle is traced in a clock-wise direction.



It is important to read the following section (as part of the pre-lab assignment) in order to familiarize yourself with the upcoming lab activity.

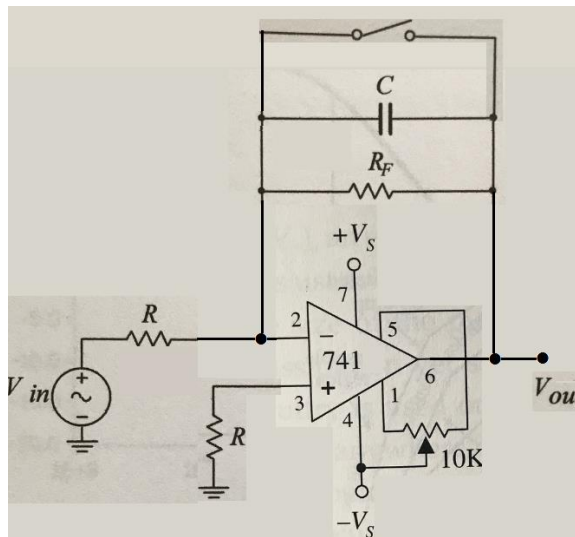
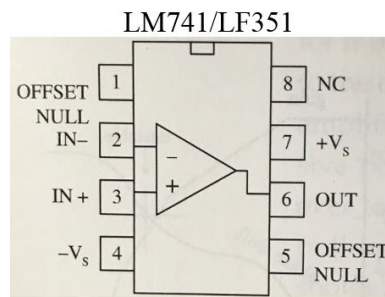
2. Lab activity

In this experiment, you will build and test an inverting integrator employing two different op amp ICs. You will experiment with the op amp voltage and current compensation. You will also build and test the three op amp circuit that you have designed in the pre-lab activity.

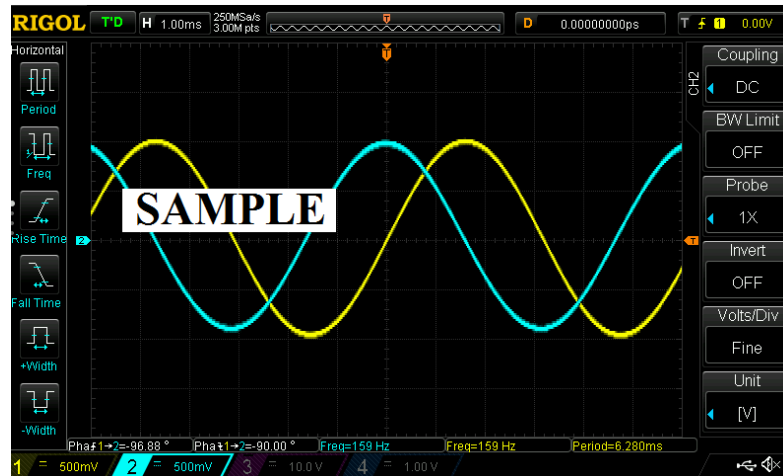
Make actual measurements of all resistor and capacitor values that you use in the following circuits and use them in your post-lab to report relative errors between the ideal circuit output and the actual circuit output. Make sure to take photographs of all circuits that you

build in this lab session. Also, you are to save all scope trace images. You will be using these photos/images in your lab report.

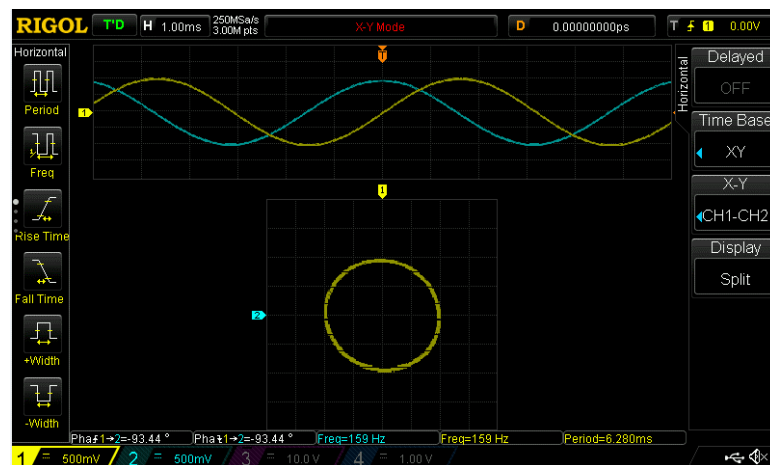
- a. Use an LM741 op amp to build the inverting integrator depicted in the figure below. Let $R_F = 1\text{M}\Omega$, $R = 10\text{k}\Omega$ and $C = 0.1\mu\text{F}$. Measure and record all component values. Set the supply voltages to $\pm V_s = \pm 15\text{V}$. The $10\text{k}\Omega$ pot should initially be set halfway so that it registers $5\text{k}\Omega$ between the edge pin and the center pin. Set the input signal to a 2V_{pp} 159Hz (or 1000 rad/sec) sine wave with zero offset. Connect the input signal to CH1 of the scope and set it to 500mV/div and 1msec/div . Connect the output of the integrator to CH2 of the scope and set to 500mV/div .



- b. Reset the op amp by discharging the capacitor by means of pressing and releasing the SPST, OFF(ON) switch, while keeping a close eye on the behavior of the output trace on the scope. Record your observations.
- c. Adjust the pot (try few turns in each direction) so that you eliminate the effect of the op amp input offset voltage (you know you have successfully archived the desired setting when the output waveform attains zero DC offset). *Make sure that the scope is displaying the frequencies of the input and output waveforms and their relative phase in the measurements bar at the bottom of the display (see the figure below). Save the scope's trace image. Your scope display should look similar to the one in the following figure.*

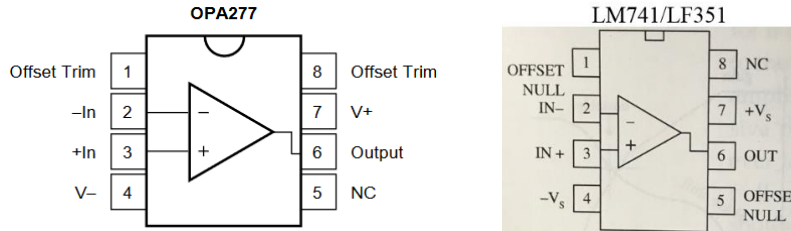


- d. Use the scope's Horizontal section "Menu" button to select the XY time base (to plot CH1 vs CH2) and display the Lissajous Pattern. Make sure that the scope is still set to 1msec/div. The pattern should be circular, indicating a 90° phase difference between the two signals. A sample image is shown below. *Restore the scope to the YT time base before proceeding to the next step.*

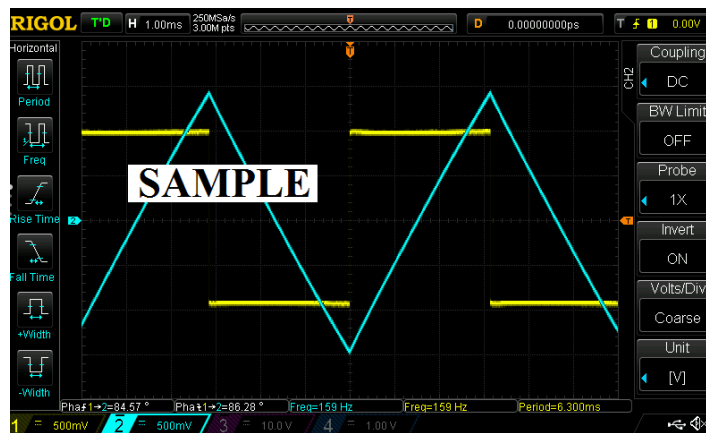


- e. What effect does shorting (to ground) the resistor connected to Pin 3 of the op amp have on the integrator's output? Un-short the resistor before proceeding to the next step.
- f. What effect does removing the feedback resistor R_F have on the integrator's output? Reinstall R_F before proceeding to the next step.
- g. What effect does removing the $10k\Omega$ pot has on the integrator's output? Do not reinstall the pot in your circuit.
- h. Switch off the supply voltage and disconnect the input. Carefully remove the LM741 (use the supplied "green" IC extraction tool) and replace it with the high-precision OPA277 op amp IC. Note: both ICs have the same pin configuration except for one of the offset

null/trim pins (but there is no need to worry since we are not using the trimming pot in the circuit anymore). Switch on the supply voltage and then connect the input signal (the 159Hz 2Vpp sine wave). Save the scope's trace image. Does the OPA277 op amp require input voltage offset trim? What effect does shorting (to ground) the resistor connected to Pin 3 have on the op amp output? Is R_F necessary? What happens if you remove it from the circuit?



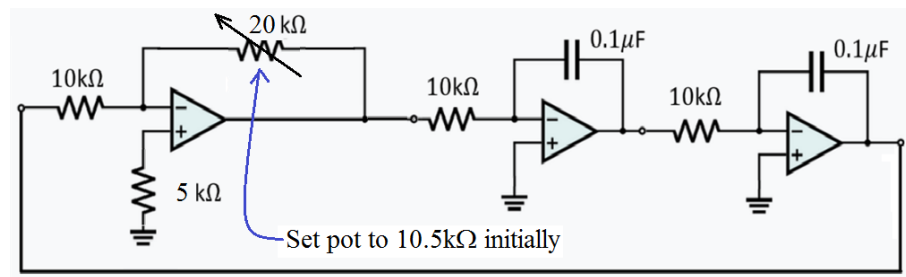
- i. Change the input signal to a square wave. Do not change the amplitude and frequency of the input wave. Set "Invert ON" on CH2 of the scope so as to undo the integrator inversion. Save the scope traces. The following is a sample image.

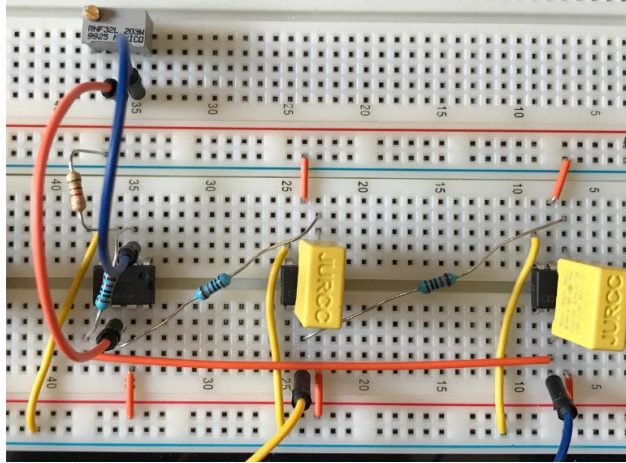


Repeat the above step with a triangular wave input.

When finished with this step, revert CH2 to "Invert OFF." Ask your instructor to remove and store the OPA277 op amp IC.

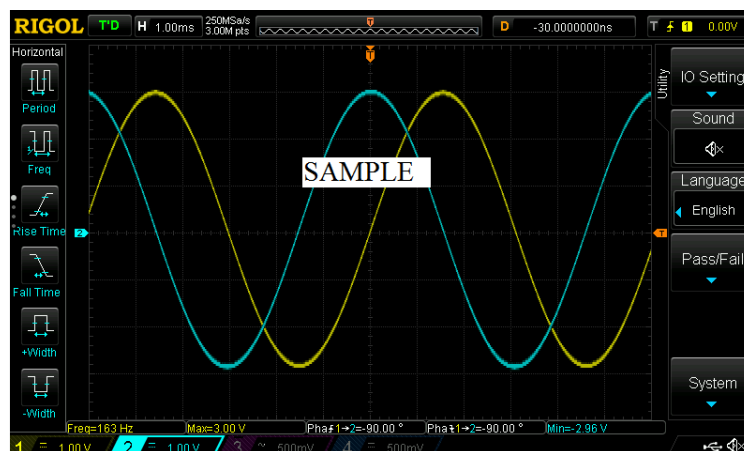
- j. Build the following sine-wave oscillator using the values specified in the figure below. Employ three LF351 op amps. Try to select the capacitors to be within 1% of each other.





Notice that for this particular circuit, there is no need for the R_F integrator resistors, because the circuit does not have an input signal (with a potential DC component). Furthermore, we can do without the $10k\Omega$ pots (this circuit does not require input op amp voltage offset adjustments). Also, the LF351 has very, very small input bias currents and the resistor between the +ve terminal and ground is not required for the integrators. (The above picture show 10% tolerance “yellow” polypropylene capacitors.)

- k. Attach the outputs of the left and right integrators to CH1 and CH2 of the scope, respectively. Set the Volt/div of each channel to 1volt/div and set the time scale to 1msec/div. Also, make sure that both waveforms have the same voltage reference on the scope. Shift the waveforms horizontally so that the output of the left integrator passes through the origin (i.e., make it look like a sine wave). Set the pot to an initial value of $10.5k\Omega$. Switch off the power supply and then back on. Wait 2-3 min for the signal to stabilize. Adjust the pot so that the left integrator waveform has a 6Volt peak-to-peak value. The pot need to be adjusted slowly to arrive at the perfect gain that sustains the oscillation; a gain above this value will saturate the signals, and below this value will decay the signals to zero. Measure and record the peak voltage of each integrator output and their frequency. Save the scope’s display image. The following is a sample plot.



- l. Measure the phase difference between the two waveforms. Do the traces look like those for a sine wave and a cosine wave? Use the scope to display the Lissajous Pattern. The pattern should be (close to) a circle, indicating a 90° phase difference between the two signals. *Restore the scope to the YT time after you are done with this step.*

As you disassemble your circuit, and in case you forgot to measure element values, carefully measure and record the actual pot, resistors and capacitors values. You will need them in your report to determine the amplification of the various stages of your circuit. You will find these measurements helpful for computing the oscillation frequency.

3. Post-lab

Write your lab report according to the required format. Analyze and discuss, in details, all results obtained from your pre-lab and your lab activity. Explain the limitations of the circuits that you have built and the sources of imperfections. Also, report the absolute relative errors (in percent) for all experimental measurement whenever the true value is known.

For the oscillator (Steps j-l), and as part of your analysis and discussion for this experiment, calculate the theoretical oscillation frequency (in Hz) based on the nominal element values. Also, calculate the frequency utilizing the measured component values. Compare your answers to the actual oscillation frequency (the one you had read off the scope). Hint: The theoretical oscillation frequency ω (rad/sec) is related to the total oscillator's closed loop gain G according to $\omega^2 = G$. (Recall that $\omega = 2\pi f$.)