

Experiment 8

Amplitude Modulation, Demodulation and Signal Reconstruction

Systems and Signals Laboratory

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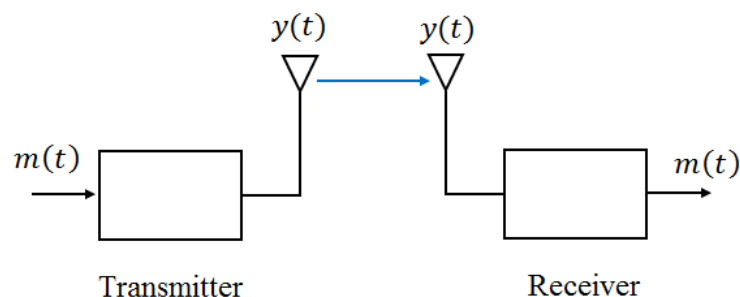
- Pre-lab
- Lab activities: Build and test an AM modulation/demodulation system and study the effect of the reconstruction filter on the quality of the recovered signal. Generate the spectra of the signals at various points in the communication system. Study the effect of phase difference between the modulation sinusoid and the demodulation sinusoid on the reconstructed signal.
- Post-lab
- Links to resources and data sheets:
 - WaveForms configuration file: [experiment8](#)
 - Krohn-Hite 3202 dual variable filter [brochure](#) and [user's manual](#)
 - [The LF351 op amp data sheet](#)
 - [The AD633 analog multiplier data sheet](#)
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Electronic instruments and components required for this experiment

- Krohn-Hite 3202 dual variable filter
- LF351 wideband operational amplifier (2)
- AD633 analog multiplier (2)
- 10k Ω (1/4 Watt, 1%) resistor (2)
- 100k Ω (1/4 Watt, 1%) resistor (2)

Background

Consider the following communication system.



There are two main factors affecting the design of a communication system. First, the transmission antenna must be of practical size. The size of the transmission antenna (L) is proportional to the wave-length of the signal,

$$L \propto \lambda = \frac{c}{f} = \frac{2\pi c}{\omega}, c \cong 3 \times 10^8 \frac{\text{m}}{\text{s}}$$

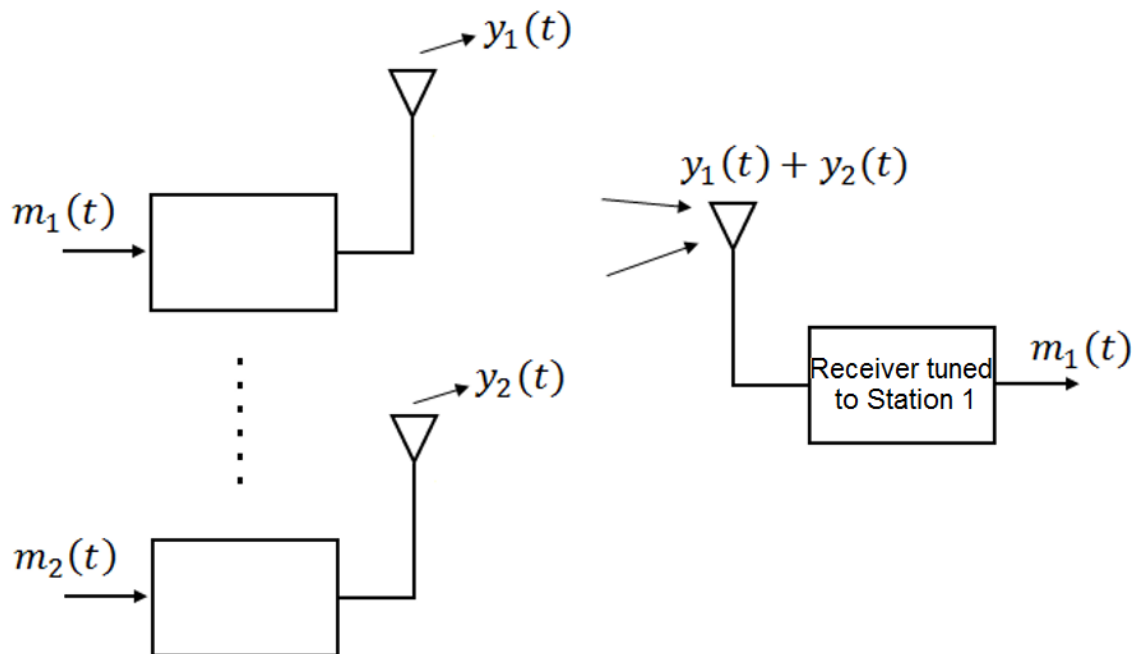
For an audio signal, $m(t)$, the frequency components are in the range $100\text{Hz} < f < 20\text{kHz}$. And the effective signal bandwidth for intelligible speech is about 5kHz.

So, if we are to transmit speech signals directly then the length of the required transmission antenna would be of the order of several kilometers:

$$L \propto \lambda > \frac{(3)10^8}{(20)10^3} \cong 15\text{km}$$

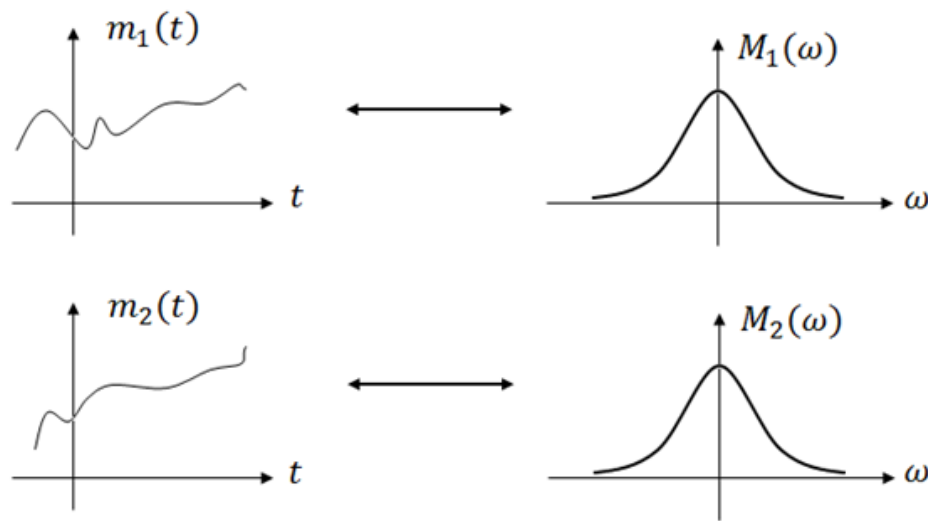
To make the transmission practical, we have to shift the signal frequency to a higher range; i.e., we need to *modulate* $m(t)$ so that it rides on a high-frequency signal $y(t)$.

Other requirements include the ability of the receiver to select one specific transmission channel from the superposition of several concurrent transmissions (by different stations), as illustrated in the following figure.



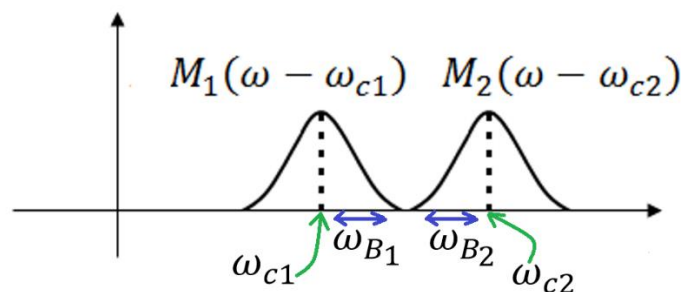
The Fourier transform is a convenient tool for designing communication systems. The frequency spectrum representation of a signal renders it easier to understand the processing and design aspects of the system, as compared to the time-domain representation. Assuming two

stations/signals $m_1(t)$ and $m_2(t)$ to be transmitted, we start by Fourier transforming the signals to obtain their frequency distribution (spectra) $M_1(\omega)$ and $M_2(\omega)$.



Next, we need to shift the spectra of $M_1(\omega)$ and $M_2(\omega)$ to high frequencies, but with minimal spectral overlap. This is achieved by applying frequency division multiplexing (FDM) as illustrated in the following figure.

Frequency Division Multiplexing (FDM)



Note: For simplicity, we assumed that $m_1(t)$ and $m_2(t)$ are even and real test signals, which leads to even and real Fourier transforms $M_1(\omega)$ and $M_2(\omega)$, respectively.

The carrier frequencies for AM radio range from 540kHz to 1,700kHz and are spaced at 10kHz intervals. Recall that the radian frequency ω is equal to $2\pi f$, where f is the Hz frequency.

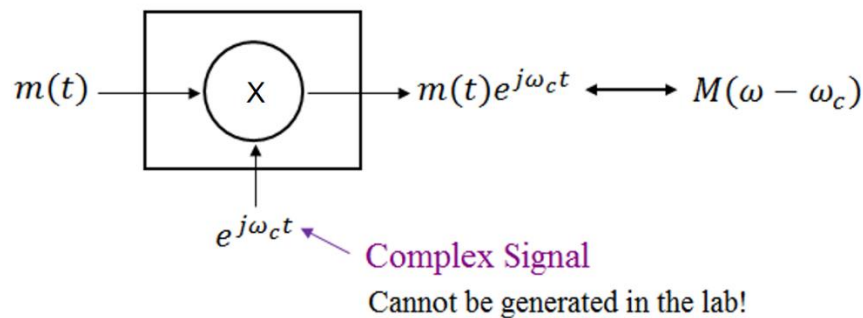
From the above figure, we notice that in order to minimize spectral overlap, we should require that the sum of the signals' (rad/sec) bandwidth to be less than the carrier frequency separation:

$$\omega_{B2} + \omega_{B1} < \omega_{c2} - \omega_{c1} \text{ or } B_2 + B_1 < f_{c2} - f_{c1}$$

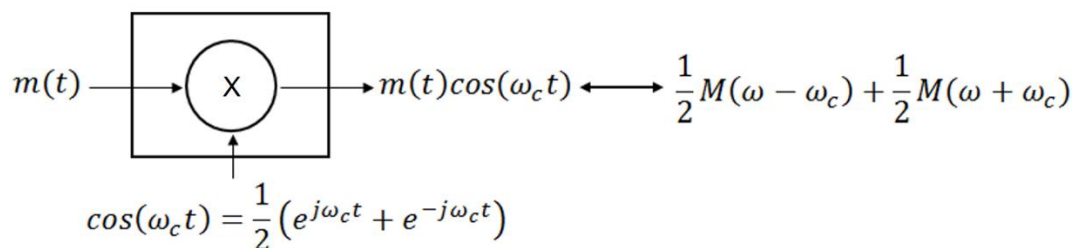
For AM radio, the intelligible signal bandwidth is about 5kHz, which would require neighboring carrier frequencies to be separated by 10kHz.

The above requirement is important so that the receiving station can employ a proper narrow band-pass filter to select the spectra of the signal of interest.

Modulation at the transmitter stage: Consider the Fourier transform pair $m(t) \leftrightarrow M(\omega)$. The frequency shift of the signal can be obtained by multiplying the time-domain signal by $e^{j\omega_c t}$ as shown below (recall the frequency-shift property of the Fourier transform).



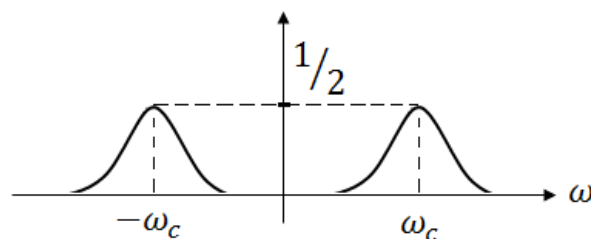
Since $e^{j\omega_c t} = \cos(\omega_c t) + j\sin(\omega_c t)$ is complex (non-physical), let us see what happens if the modulator used $\cos(\omega_c t)$ as the modulation signal.



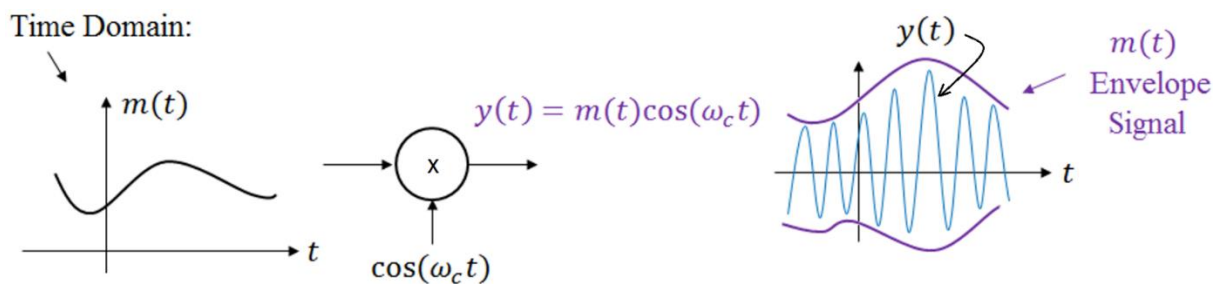
So, the transmitted signal has a duplicated spectrum centered at $\pm\omega_c$

$$y(t) \leftrightarrow \frac{1}{2}M(\omega - \omega_c) + \frac{1}{2}M(\omega + \omega_c)$$

with uniform attenuation of 50% for each component (refer to the spectral plot shown below).



The following figure depicts the modulation operation in the time domain:



Now, having received the modulated and transmitted signal $y(t)$ at the receiver, how can one extract $m(t)$?

$$y(t) = m(t) \cos(\omega_c t) \rightarrow \boxed{?} \rightarrow m(t)$$

Let us assume that the receiver can generate a $\cos(\omega_c t)$ that is in phase with the transmitter's modulation sinusoid. Multiplying $y(t)$ by this sinusoid generates:

The diagram shows the demodulation process. The received signal $y(t)$ enters a circular multiplier block from the left. A locally generated carrier signal $\cos(\omega_c t)$ enters the multiplier from the bottom. The output of the multiplier is the demodulated signal $e(t)$, which is given by the equation $e(t) = y(t) \cos(\omega_c t) = m(t) \cos^2(\omega_c t)$.

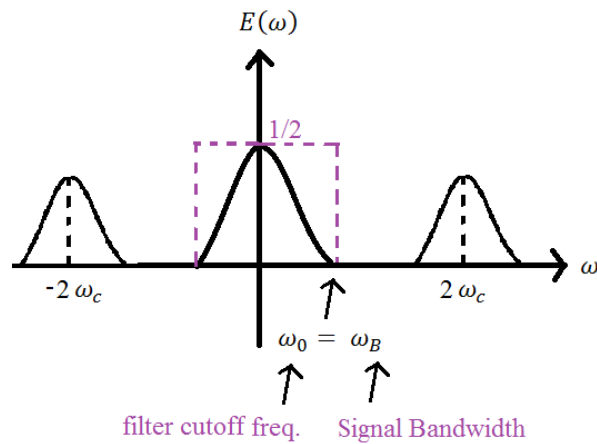
Using the trigonometric identity: $\cos^2(x) = \frac{1}{2} (1 + \cos 2x)$, we may write the *demodulated signal* $e(t)$ as

$$e(t) = \frac{1}{2} m(t) + \frac{1}{2} m(t) \cos(2\omega_c t)$$

and write its corresponding Fourier transform as

$$E(\omega) = \frac{1}{2} M(\omega) + \frac{1}{4} [M(\omega + 2\omega_c) + M(\omega - 2\omega_c)]$$

The following figure depicts the magnitude spectrum of $E(\omega)$,

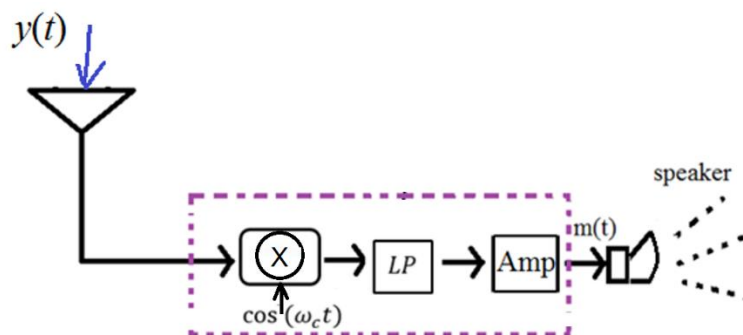


The above spectrum suggests that $\frac{1}{2} M(\omega)$ can be extracted using an ideal low-pass filter whose cutoff frequency is equal to the bandwidth of $m(t)$.

$$E(\omega) \rightarrow \boxed{LP} \rightarrow \frac{1}{2} M(\omega)$$

$$e(t) \rightarrow \boxed{LP} \rightarrow \frac{1}{2} m(t)$$

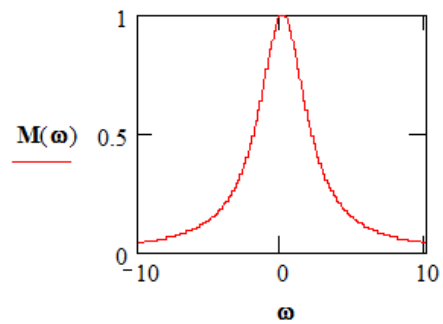
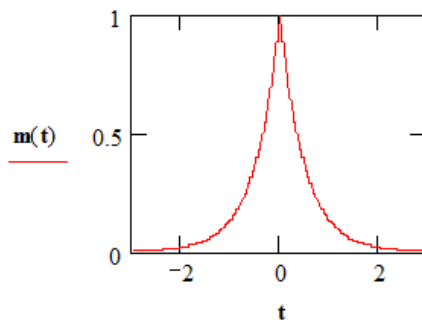
The above analysis suggests the following receiver circuit [ω_c represents the carrier frequency in rad/sec],



Example. The following is a Mathcad simulation that validates the above communication system for the even signal $m(t) = e^{-2|t|}$. It assumes that the modulation (and demodulation) signal is $\cos(\omega_c t)$, where $\omega_c = 40$ rad/sec.

$$m(t) := e^{-2 \cdot |t|}$$

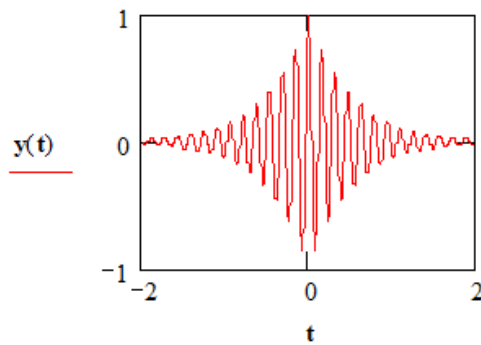
$$M(\omega) := m(t) \text{ fourier, } t \rightarrow \frac{4}{(4 + \omega^2)}$$



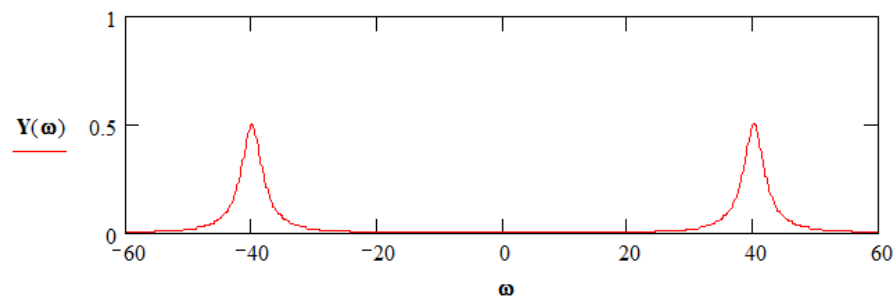
Note that bandwidth of $m(t)$ is about 10 rad/sec +

Modulation

$$y(t) := m(t) \cdot \cos(40 \cdot t)$$

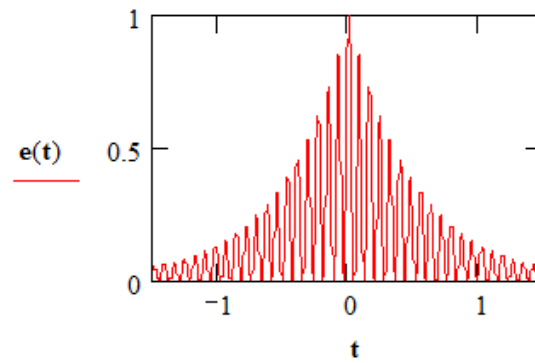


$$Y(\omega) := y(t) \text{ fourier, } t \rightarrow \frac{2}{[4 + (\omega - 40)^2]} + \frac{2}{[4 + (\omega + 40)^2]}$$

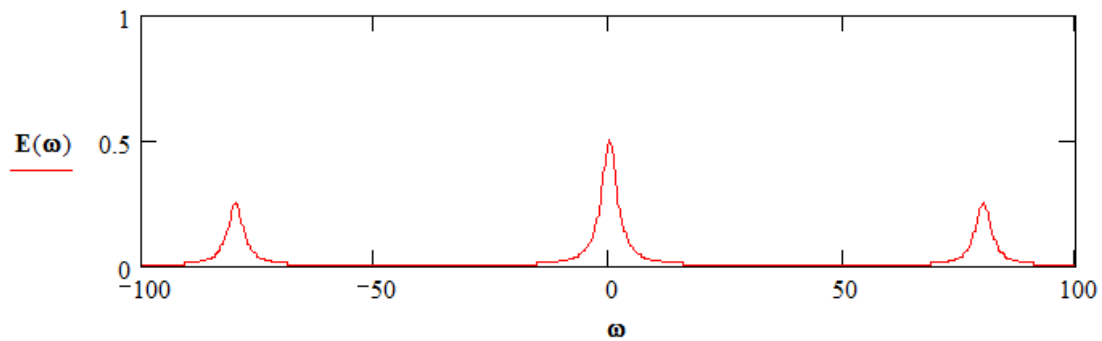


Demodulation

$$e(t) := y(t) \cdot \cos(40 \cdot t)$$

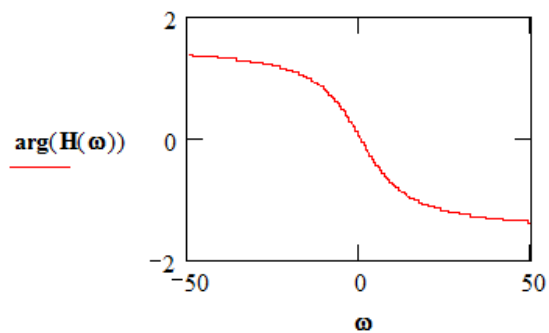
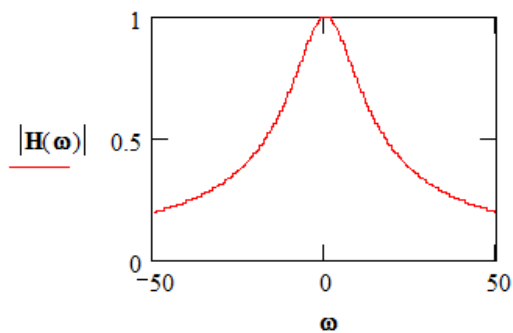


$$E(\omega) := \text{fourier}(e(t), t) \rightarrow \frac{1}{\sqrt{4 + (\omega - 80)^2}} + \frac{2}{(4 + \omega^2)} + \frac{1}{\sqrt{4 + (\omega + 80)^2}}$$



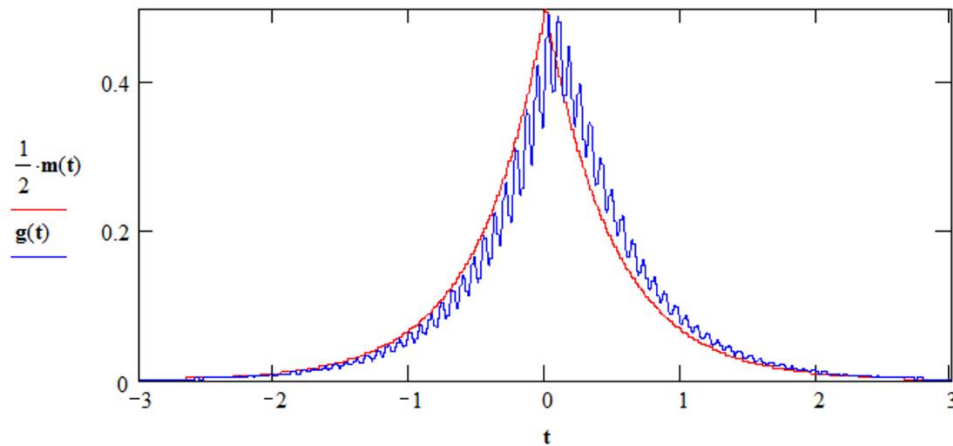
Recovering the signal using a first-order low-pass Butterworth filter with 10 rad/sec cutoff frequency:

$$\text{Low pass filter } H(\omega) := \frac{10}{10 + j \cdot \omega}$$

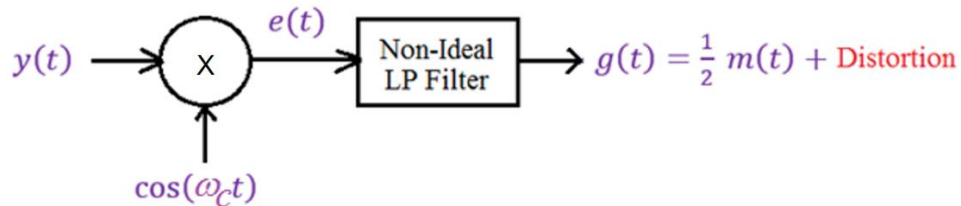
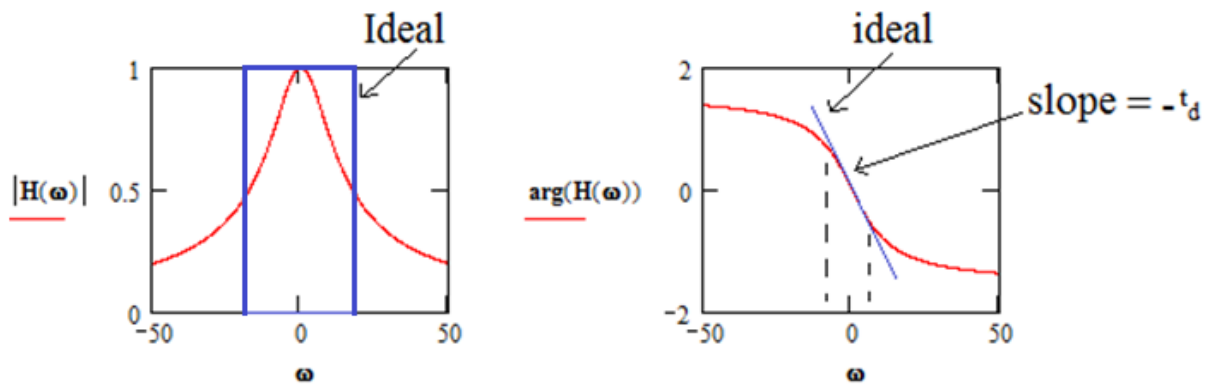


$$g(t) := H(\omega) \cdot E(\omega) \text{ invfourier, } \omega \rightarrow \frac{15}{3272} \cdot \exp(2 \cdot t - 80 \cdot i \cdot t) \cdot \Phi(-t) + \frac{25}{818} \cdot i \cdot \exp(2 \cdot t - 80 \cdot i \cdot t) \cdot \Phi(-t) + \dots$$

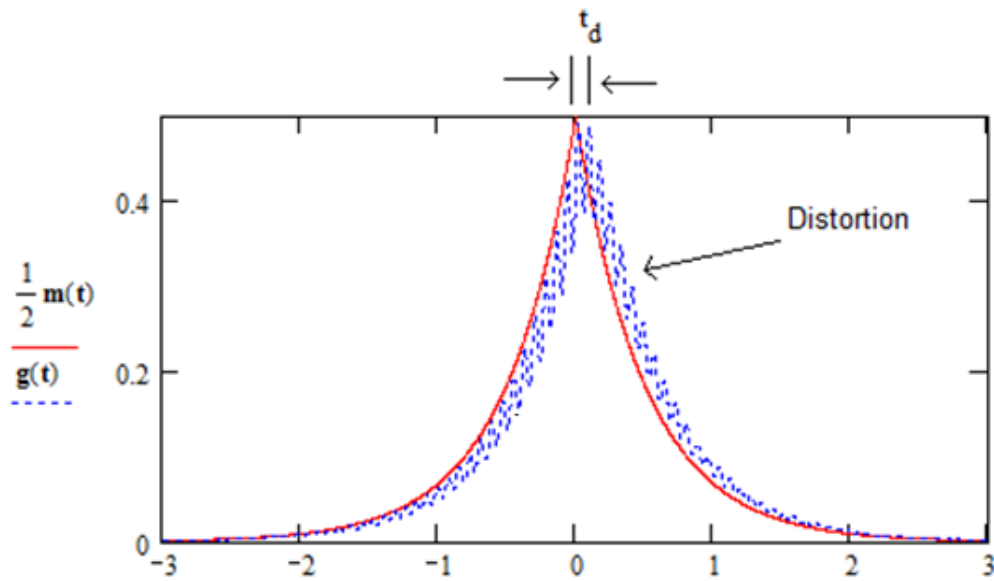
$$t := -3, -2.999 \dots 3$$



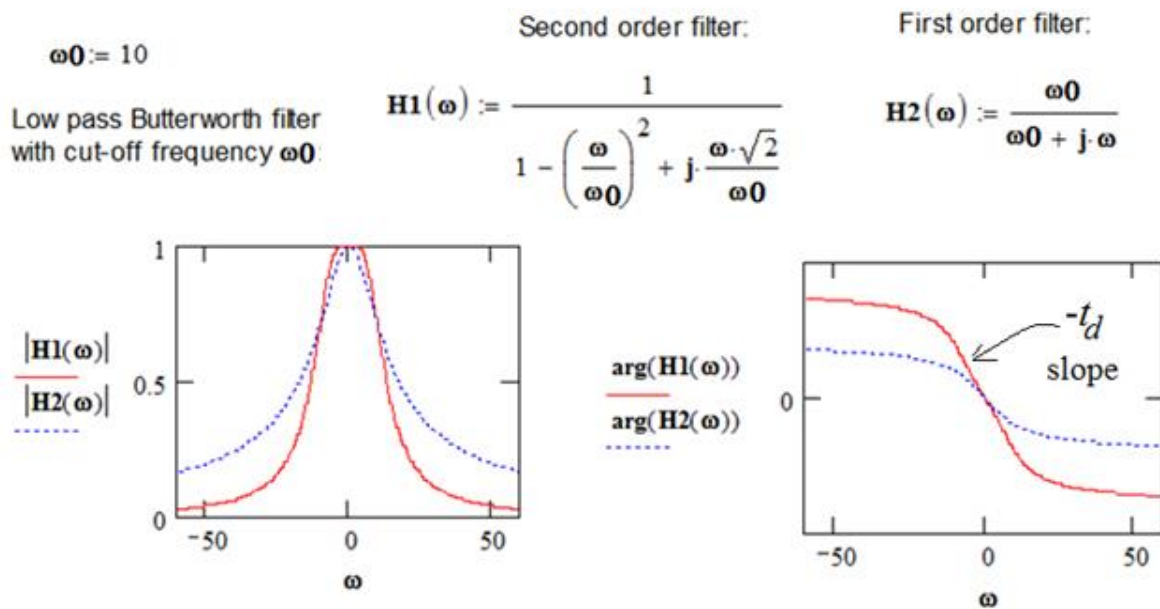
When a non-ideal low-pass filter is used, the output signal will be distorted, as can be seen from the above simulation with the first-order low-pass filter. The following figure compares the magnitude and phase response of an ideal “brick wall” filter and to that of a first-order low-pass Butterworth filter.



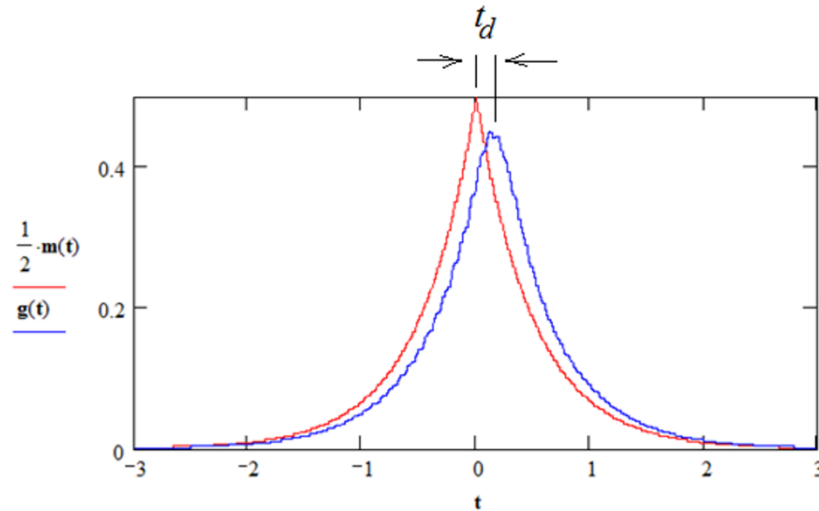
The slope of the phase response in the pass-band region of the filter, $-t_d$, represents the delay of the filter’s output and is depicted in the following figure.



In the following, the above simulations are repeated employing a 2nd-order low-pass Butterworth filter with 10 rad/sec cutoff frequency. The following figure illustrates the improved magnitude response of the 2nd-order filter.

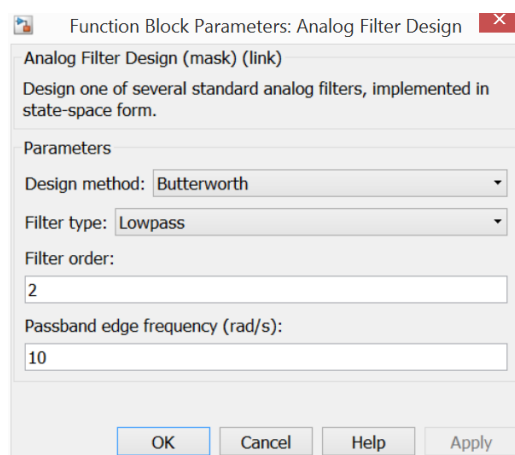
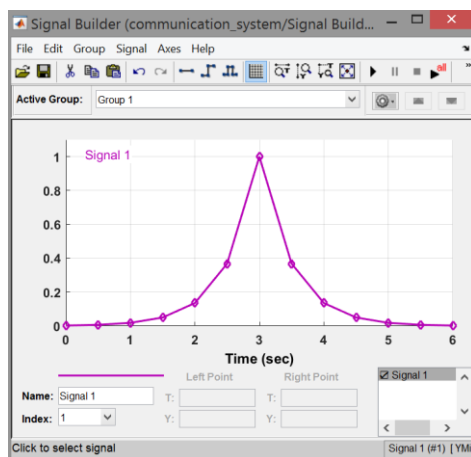
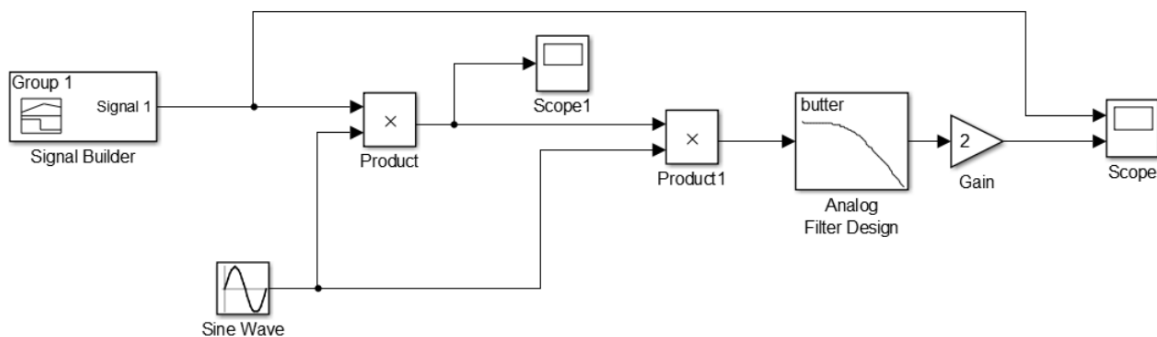


The following figure shows the reconstructed (demodulated and low-pass filtered) signal. The 2nd-order filter leads to significantly lower distortion.

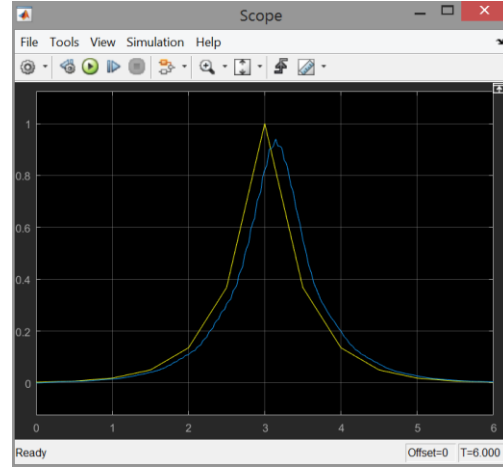
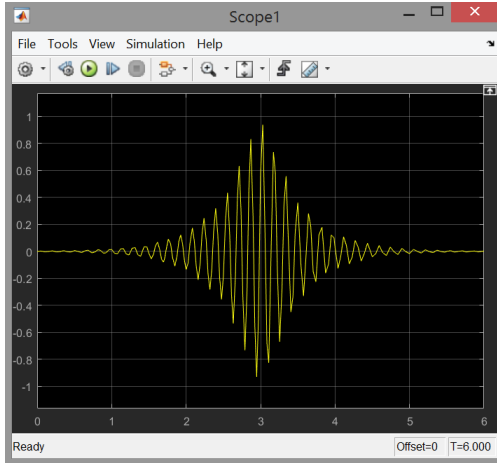


The following pictures outline a repeat of the above simulation of the communication system using Matlab's Simulink (employing a 2nd-order filter with cut-off frequency $\omega_0 = 10$ and dc gain of 2):

communication_systemR2015b

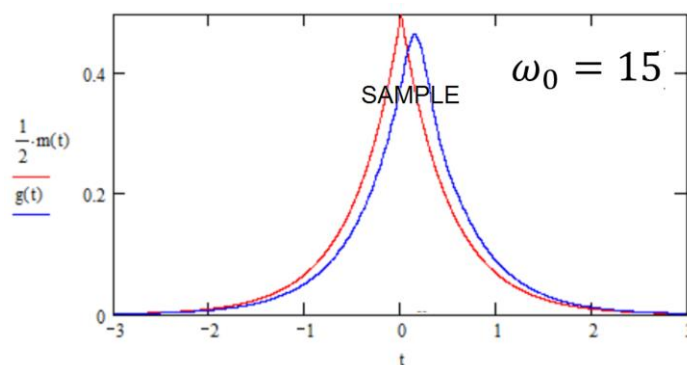
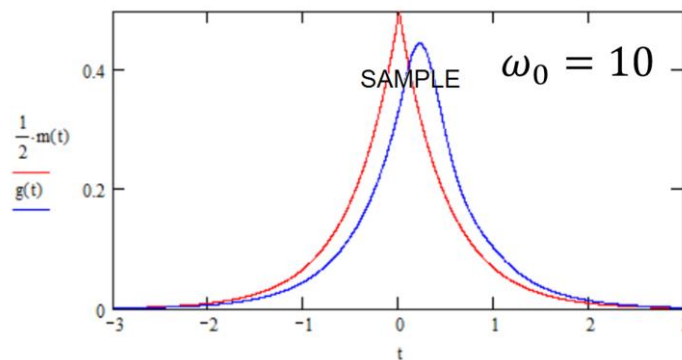


The modulated and reconstructed signals are depicted in the following two figures, respectively.

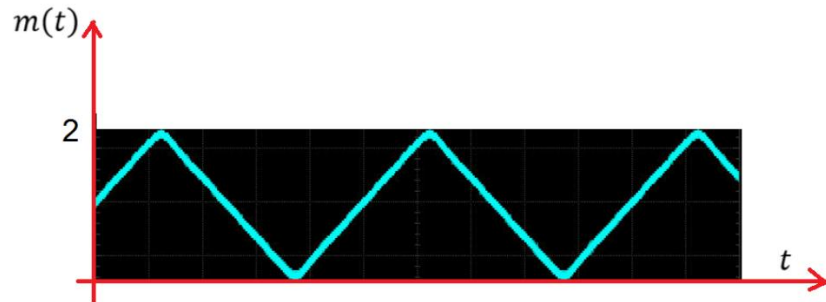


1. Pre-Lab

- Study the above “Background” section, which should help you complete the following steps.
- Repeat (employ Mathcad or Simulink) the above communication system simulation employing a 3rd-order low-pass Butterworth filter with $\omega_0 = 10$. Repeat with $\omega_0 = 15$. The simulation results (using Mathcad) are shown in the following two figures.



- c. Assume a signal $m(t)$ is modulated by $\cos(\omega_c t)$ and then demodulated by $\cos(\omega_c t + \theta)$. Show that the reconstructed signal (at the output of an ideal low-pass filter) is given by $\frac{1}{2} \cos(\theta) m(t)$. Discuss the effect of θ on the reconstructed signal.
- d. Employ the [Table of Fourier Series](#) to determine and sketch the magnitude spectrum $[C_n \text{ versus frequency (Hz)}]$ for a 2V max amplitude, 0V min amplitude and a 1kHz frequency triangular wave, $m(t)$.

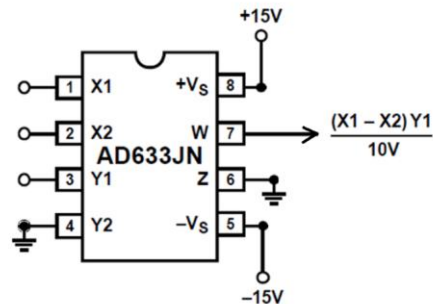
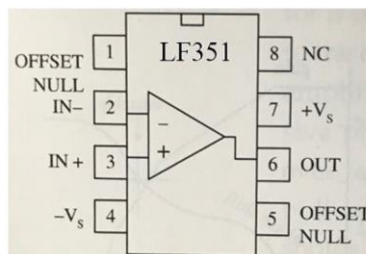


Also, construct the magnitude spectrum of the modulated signal, $y(t) = m(t)\cos(2\pi(20,000)t)$. Here, the modulation signal is a 2Vpp, 20kHz sinusoid.

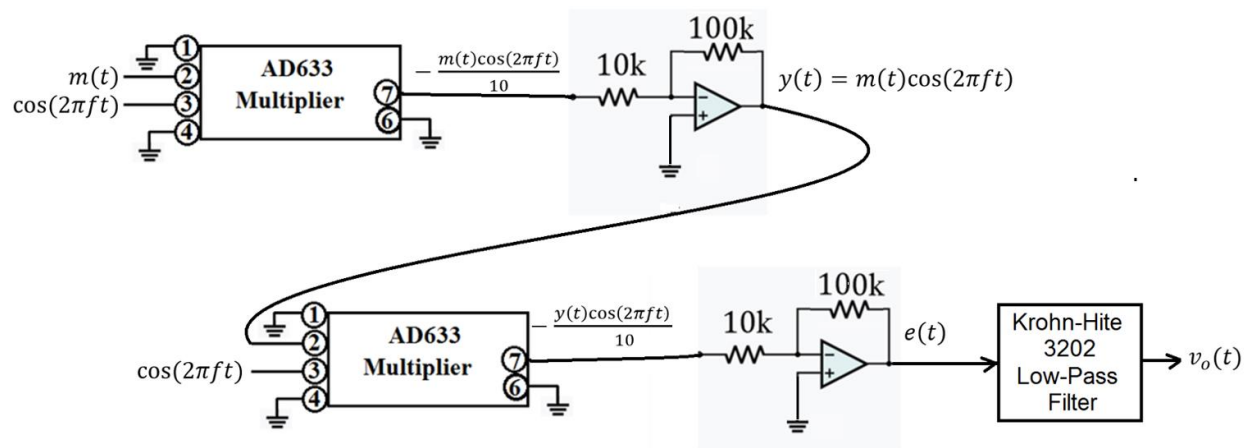
- e. Search your laptop for the folder that contains the configuration file (for WaveForms) “default.dwf3work” and then download “[experiment8](#)” configuration file and place it inside that folder. You will need this file for the Lab activity.

It is important to read the following section (as part of the pre-lab assignment) in order to familiarize yourself with the upcoming lab activity.

2. Lab-Activity

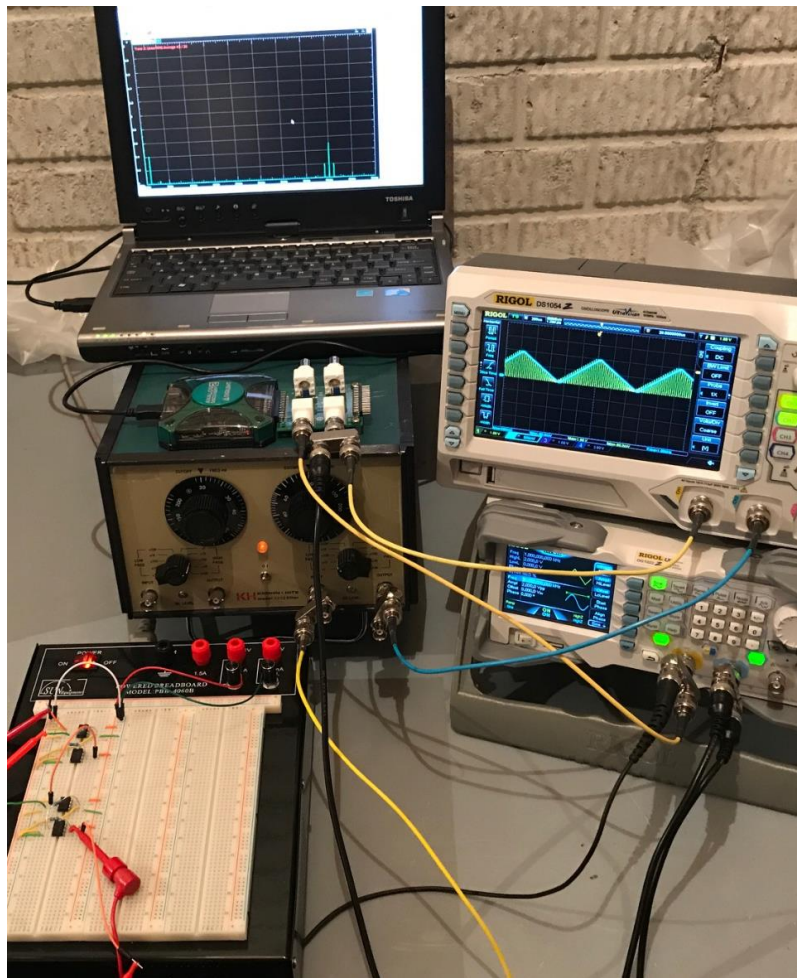


- a. Use the breadboard to build the AM communication system/circuit shown below.



Make sure that all seven chips have the proper $\pm 15V$ supply connections

The following is a photograph of the complete setup for this experiment.



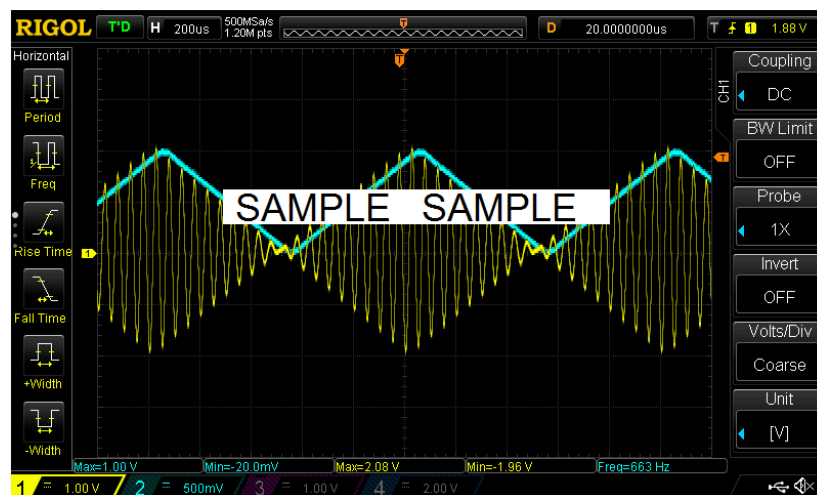
- b. Let $m(t)$ be a 2V amplitude, 1kHz triangular signal generated on CH1 of the RIGOL waveform generator. Add a bias to $m(t)$ such that its minimum amplitude is zero and its maximum amplitude is 2V. Employ BNC-BNC splitters and proper cables to connect this signal to pin 2 of the upper AD633 analog multiplier, and to CH1 of the Digilent AD2 module.

Generate a 2Vpp, 20kHz sinusoid on CH2 of the RIGOL waveform generator and connect it to pin 3 of the upper multiplier and pin 3 of the lower AD633 multiplier. Use the “Align Phase” button to align the two signals on CH1 and CH2 of the waveform generator.

Connect the modulated signal $y(t)$ to CH2 of the AD2 module, to CH1 of the RIGOL scope and to pin 2 of the lower multiplier.

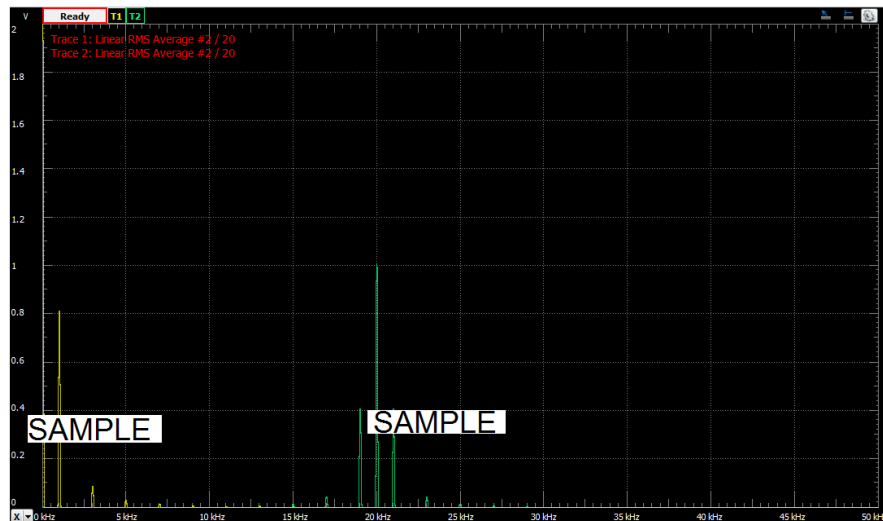
Connect the demodulated signal $e(t)$ to the input of the Krohn-Hite 3202 filter. Connect the output of the filter to CH2 of the RIGOL scope. Set the filter to low-pass with 10kHz cutoff frequency; the filter is now configured for a 4th order Butterworth filter with 10kHz cutoff frequency, and is supposed to pass all significant spectral components of $m(t)$.

Set CH1 and CH2 of the RIGOL scope to 1V/div and 500mV/div, respectively. Set the time scale to 200 μ sec/div. Configure the information bar of the scope to display the max and min amplitudes of the signals on CH1 and CH2, and the frequency of the signal on CH2. Shift scope traces so that they look like the ones shown in the following sample image. Save the scope's display image.

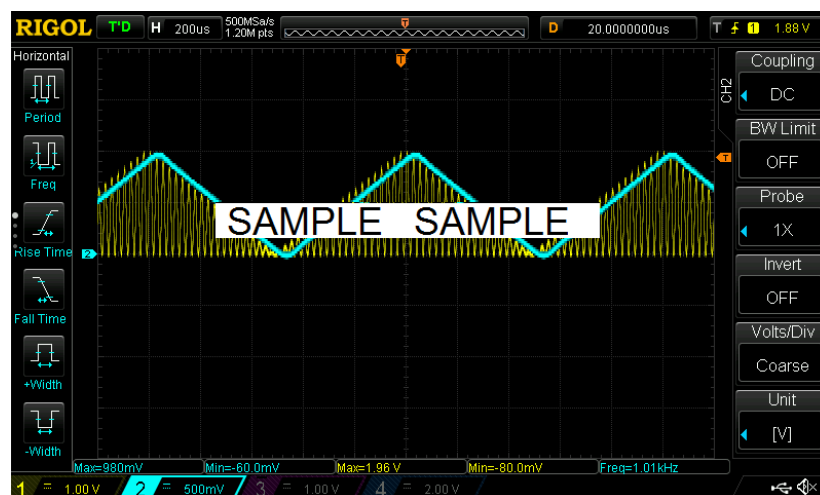


Note that the reconstructed signal (on CH2 of the scope) is very close to the input triangular signal $m(t)$, except for a 50% uniform amplitude attenuation and a slight phase shift. The phase shift is due to the low-pass filter.

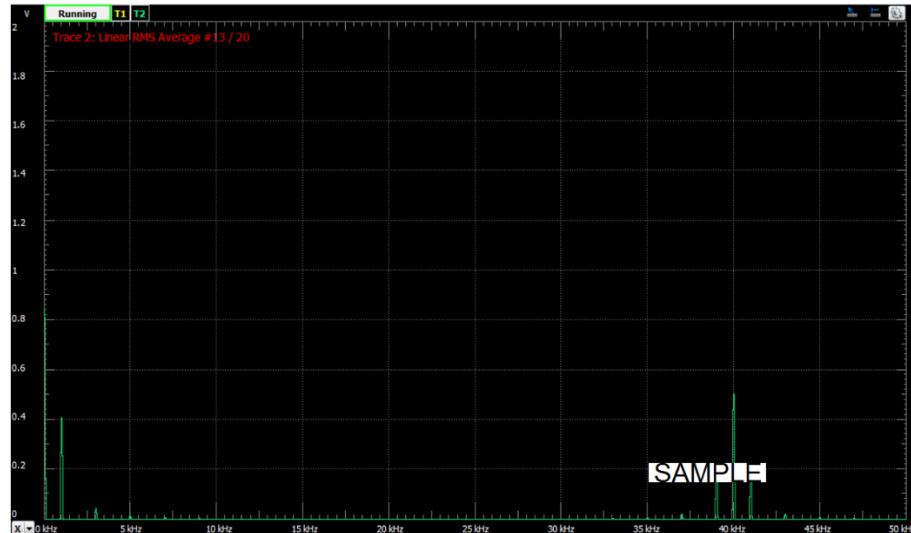
- c. Connect the AD2 module to your laptop and run the WaveForms software. Select “experiment8” as configuration file when prompted. Click on the “Run” icon of WaveForms. You should see the following spectra of the triangular wave and the modulated wave. The left half of the spectrum of the triangular wave (corresponding to negative frequencies) is not displayed. Capture and save the WaveForm’s screen.



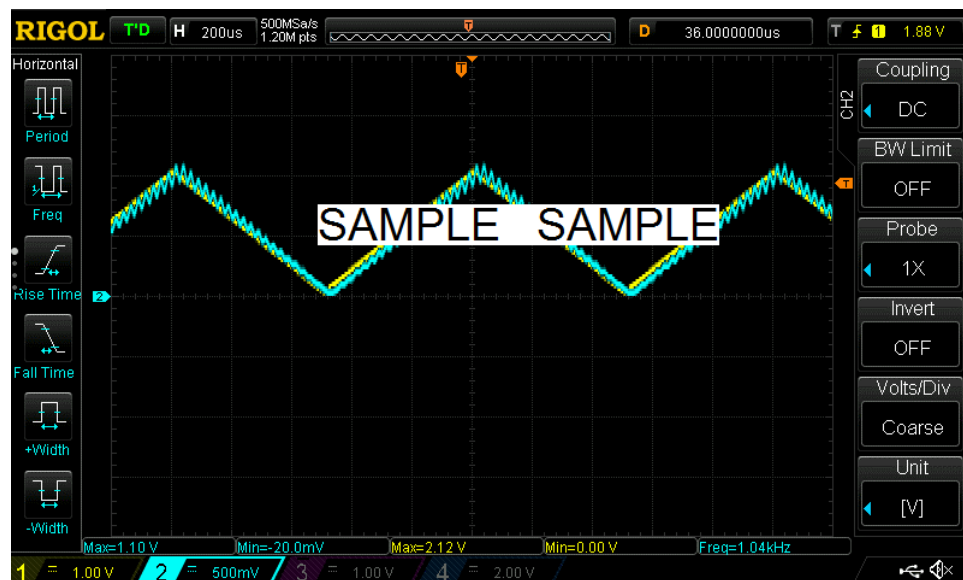
- d. Measure the peaks of the spikes of the input (yellow) and modulated (green) signals. Include the amplitude of the spike at 0Hz (dc). In your lab report, you are to compare these values to the theoretical ones. Notice how the spectrum of the modulated signal $y(t)$ is a scaled (attenuated by 50%) and shifted (centered at the modulation frequency of 20kHz) version of the input signal, $m(t)$.
- e. Switch the input of CH2 of the AD2 module and CH1 of the RIGOL scope from the modulated signal $y(t)$ to the demodulated signal, $e(t)$. Save the scope image (see the sample below).



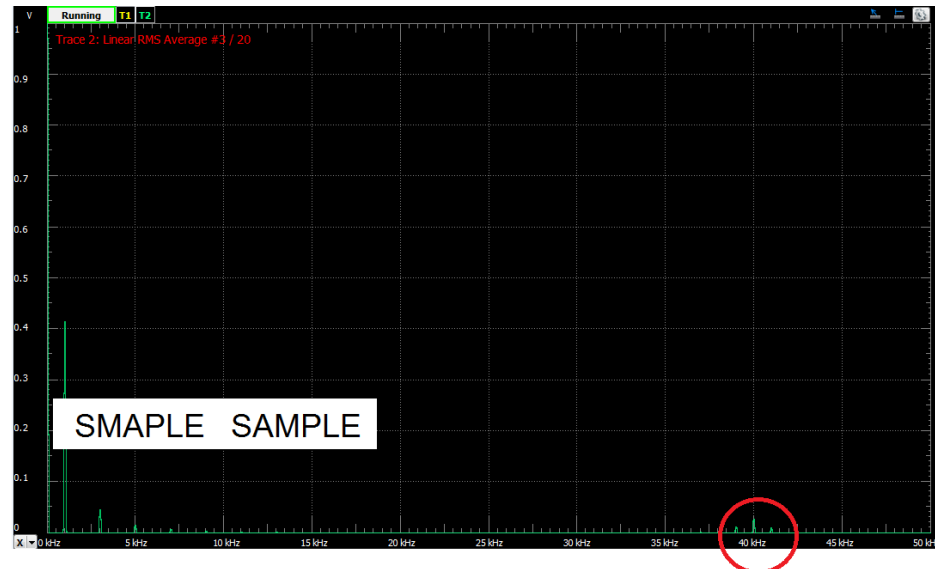
Disable the (yellow) spectrum of $m(t)$ by unchecking the small box to the left of “Trace 1” (located on the right hand side of the WaveForms display). The laptop’s screen should now display the spectrum of the demodulated signal, as shown below. Capture and save the display. Notice how two scaled copies of the spectrum of $m(t)$ are showing centered at zero and 40kHz frequencies. Measure the peak values of the spectral components.



- f. Connect CH2 of the AD2 module to the output of the filter (keep CH1 of the AD2 disabled). Change the “Top” value inside the “Magnitude” box of WaveForms to 1V. Connect CH1 of the RIGOL scope to the input signal, $m(t)$. Save the RIGOL scope’s display and capture the WaveForms screen (which should now be displaying the spectrum of the reconstructed/filtered signal) for each one of the following filter cutoff frequencies: 2kHz, 4kHz, 10kHz, 20kHz and 30kHz. The following sample scope image depicts the system’s input and out signals for the 20kHz cutoff frequency.



The spectrum of the filtered signal $v_o(t)$ is depicted in the figure below. The filter passes a fraction of the spectral image that is centered at 40kHz which leads to distortion in the reconstructed signal, $v_o(t)$.



- g. Set the cutoff frequency of the filter to 2kHz. Disconnect the triangular wave input. Connect the input of the circuit, $m(t)$, to the audio jack of your smartphone. Connect the output of the filter to a speaker. Change the modulation frequency (on CH2 of the RIGOL waveform generator) to 10kHz. Use the RIGOL scope to monitor $m(t)$ on CH1 (set to 100mV/div) and the output of the filter on CH2 (set it to 50mV/div). Set the time scale of the scope to 500 μ sec/div.

Play piano or trumpet music (you can get it through YouTube) on your smartphone and listen to it on the external speaker. If the sound is too high, lower your smartphone's volume.

How does the sound change as you slowly sweep the filter cutoff frequency from 2kHz to 20kHz? Document your observations. By simply listening to the speaker, while monitoring the scope traces, estimate the optimal value of the cutoff frequency that minimizes distortion. Hint: you may try single trace scope capture to see portions of the reconstructed signal more clearly. Watch sample [movie](#).

- h. The human ear can't hear above 20kHz. So, we can view the ear as a low-pass filter. Also, the mechanical vibration nature of the speaker's membrane makes it attenuate very high frequencies, rendering the speaker as another low-pass filter. In other words, the Krohn-Hite filter can be eliminated and you would still hear audible music!

Confirm this phenomenon by connecting the demodulated signal, $e(t)$, directly to the speaker. What happens if the modulation/demodulation sinusoid frequency is set below 7kHz? Why?

- i. Take photos of your circuit and the complete experimental set up. Ask your lab instructor to take a photo of you and your partner with your experimental setup showing in the background. Use these photos in your lab report.
- j. In this step you will experimentally study the effect of a phase shift θ between the modulation signal and the demodulation signal. The second half of the Krohn-Hite filter will be used to introduce the phase shift θ .

Disconnect all cables from the filter, from the RIGOL scope and from the RIGOL waveform generator. You will not be using the AD2 in this step. Connect BNC-BNC splitters to the input and output connectors of the left side filter of the Krohn-Hite instrument. Set the left side filter to low-pass with cutoff frequency of 200kHz. This filter will be used to set θ . Set the right side filter to low-pass at 10kHz cutoff frequency.

Switch on the four channels of the scope and set the scales to 1V/div for CH1, CH2 and CH3. Set CH4 to 500mV/div. Set the scope to 100 μ sec/div. Shift CH1 and CH2 traces to the center of the top half of the display and make both channels share the same reference. Shift CH3 and CH4 traces to the center of the lower half of the display and make both channels share the same reference. Display the phase difference between CH1 and CH2 (i.e., display θ) and the peak-to-peak voltage of CH3 and CH4 in the scope's measurements bar. Set scope triggering to CH3. Make sure that all 4 channels are set to "Invert: OFF."

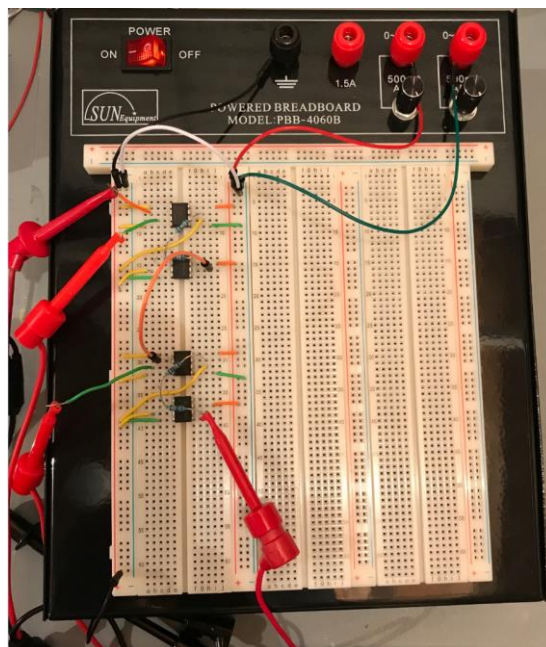
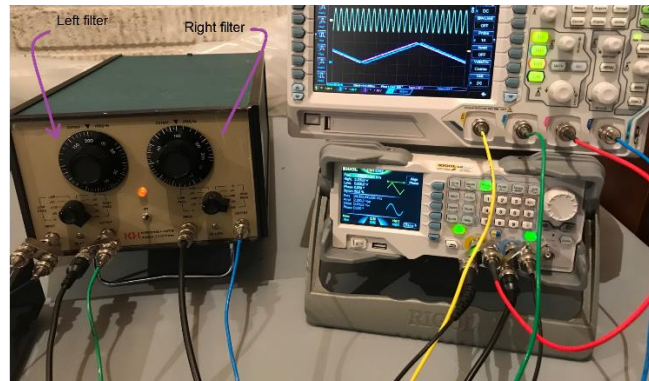
Set CH1 of the RIGOL waveform generator to 1kHz triangular wave with 2V max and 0V min voltages, as in Step b. CH1 outputs $m(t)$. Set CH2 to a sinusoid with 2Vpp at 20 kHz frequency. This is the modulation/demodulation signal. Align the phases of CH1 and CH2. Connect a BNC splitter to CH1 and another to CH2.

Connect CH1 of the waveform generator to the input of the circuit (pin 2 of the first multiplier) and to CH3 of the scope.

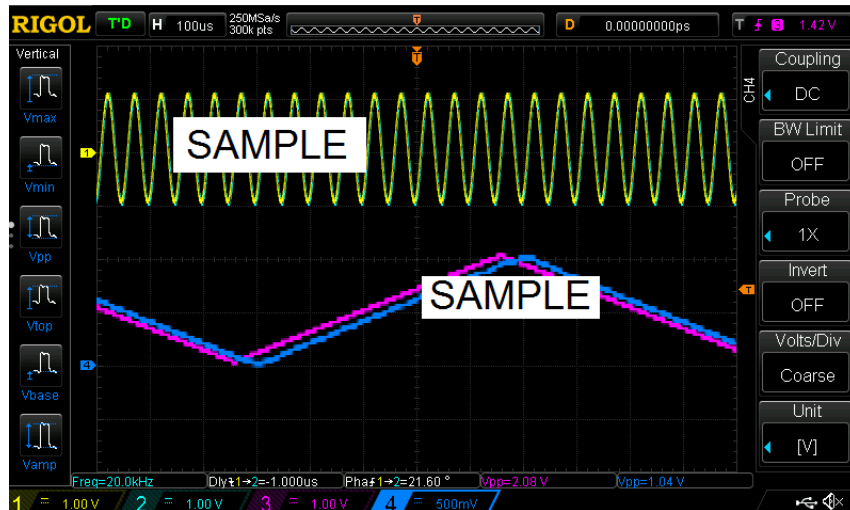
Connect CH2 of the waveform generator to pin 3 of the first multiplier and to the input of the left filter.

Connect the input of the left filter (at the BNC splitter) to CH1 of the scope. Connect the output of the left filter (at the BNC splitter) to CH2 of the scope and to pin 3 of the second multiplier.

Connect the output of the second inverting amplifier, $e(t)$ to the input of the right filter. Connect the output of the right filter to CH4 of the scope. The following are pictures of the setup and its associated connections.



Apply power to your circuit and switch the filter on. If all goes well, you should see the following traces on the scope.



With the low-pass (left) filter setting at 200kHz cutoff frequency, θ is very small (about 14°) and the reconstructed signal is given by $v_o(t) = \frac{1}{2} \cos(14^\circ) m(t) \cong 0.49m(t)$.

- Record the peak-to-peak voltage of $m(t)$ and $v_o(t)$ associated with this phase difference. Save the scopes' display image.
- Change the cutoff frequency of the left filter to 60kHz. This should increase θ to about 50° . Record the value of θ and the peak-to-peak voltage of $m(t)$ and $v_o(t)$. Save the scopes' display image.
- Decrease the cutoff frequency of the left filter until you reach $\theta \cong 75^\circ$. Record the peak-to-peak voltages of $m(t)$ and $v_o(t)$. Save the scopes' display image. Repeat for $\theta \cong 90^\circ$.
- What happens if the left filter cutoff frequency goes below 35kHz? Explain.

Post-lab

Write your lab report according to the required format. Analyze and discuss, in details, all results obtained from your pre-lab and your lab activity. Compare the theoretical, simulation and experimental performance of the filter. Report the absolute relative errors (in percent) for all experimental measurement whenever the true (ideal) value is known.